



Anyagfizika

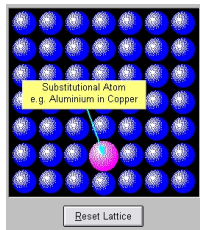
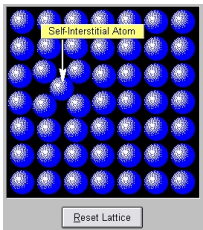
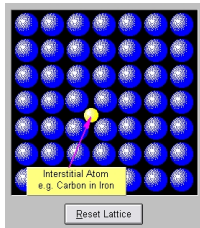
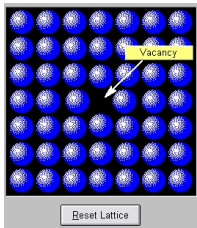
Kristályhibák

Groma István

ELTE

March 10, 2022





$$S(U, V, N) \longrightarrow F(T, V) = U - TS$$

$$dU = TdS - pdV$$

$$dF = SdT - pdV$$

Rendszer + termosztat = zárt

$$dS_r + dS_t \geq 0$$

$$dU_r + dU_t = 0$$

$$dS = \frac{dU}{T} + \frac{p}{T}dV$$

innen mivel itt $dV = 0$

$$dS_s - \frac{dU_s}{T} \geq 0 \longrightarrow dU_s - TdS_s \leq 0$$

$$dT = 0, \quad dF = dU - TdS - SdT \leq 0$$

Egyensúlyi vakanciakonzentráció
Gibbs potenciál

$$G(T, p) = U - TS + pV$$

n vakancia N atom

$$\Delta U = \epsilon_0 n$$

$$\Delta V = v_0 n$$

Entrópia

$$S^{config} = k_B \ln \binom{N+n}{n} = k_B \ln \frac{(N+n)!}{N!n!}$$

Stirling-formula

$$\ln n! \approx n \ln n$$

Így

$$S^{config} = k_B [(N+n) \ln(N+n) - N \ln N - n \ln n]$$

Innen

$$\Delta G = \epsilon_0 n + p v_0 n - k_B T [(N+n) \ln(N+n) - N \ln N - n \ln n]$$

Egyensúly koncentráció

$$\frac{\partial \Delta G}{\partial n} = 0$$

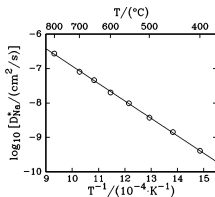
$$\epsilon_0 + p v_0 - k_B T [\ln(N + n) - \ln n] = 0$$

$$\frac{n}{N + n} = e^{-\frac{\epsilon_0 + p v_0}{k_B T}}$$

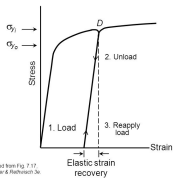
$$N \gg n$$

$$n = N e^{-\frac{\epsilon_0 + p v_0}{k_B T}}$$

Arrhenius plot (nátrium)

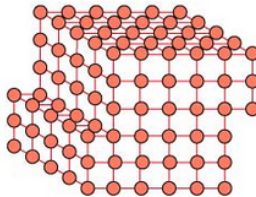
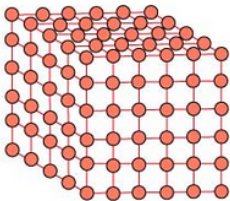


Elastic Strain Recovery



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Ideális nyírás



Ideális nyírás

$$F(x) = F_0 \left| \sin \left(\frac{\pi}{a} x \right) \right| = \sigma a^2$$

Kis deformáció

$$\gamma = \frac{x}{a}, \quad \sigma = \mu \frac{x}{a}$$

Így

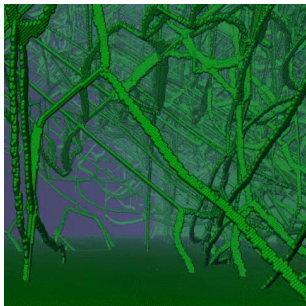
$$F_0 = \mu \frac{a^2}{\pi}$$

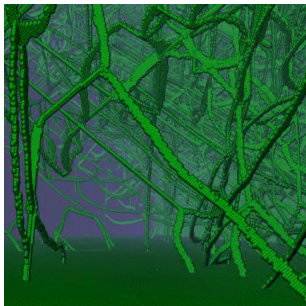
Innen

$$\sigma_f \cong \frac{\mu}{\pi} \cong 30 \text{ GPa}$$

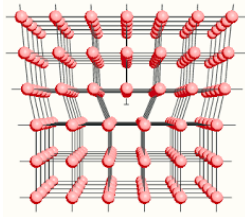
Kísérlet

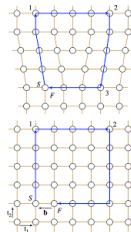
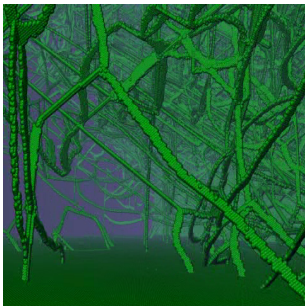
$$\sigma_f \cong 100 \text{ MPa}$$



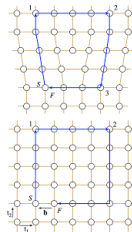
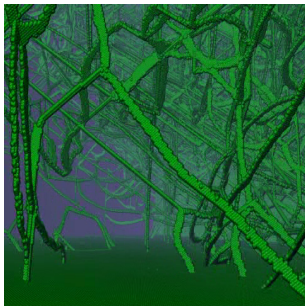


Polányi, Orován, Taylor (1934)



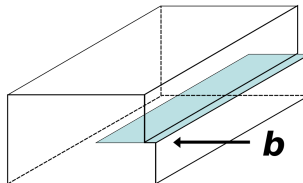


Burgers vector

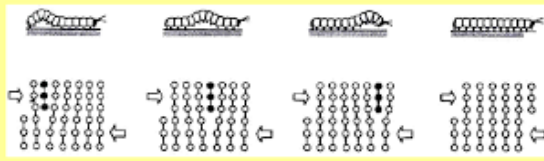


Burgers vector

Vito Volterra (1905)

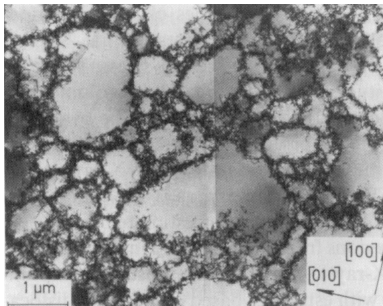


Az elcsúszás nem egyszerre megy végbe
Ez csak akkor lehetséges, ha a kristály
tartalmaz **egyméretű rácshibákat,**
diszlokációkat.

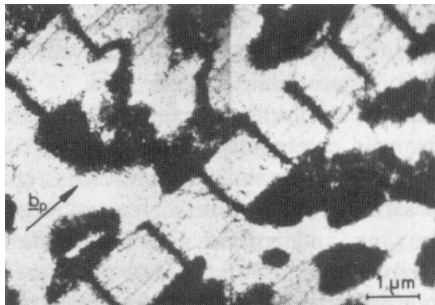


Csúszás: atomok száma megmarad

Mászás: atomok száma nem marad meg, magas hőmérséklet kell

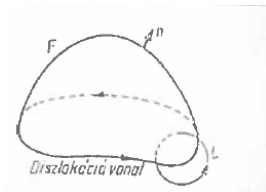


Cella szerkezet



PSB szerkezet

diszlokáció sűrűség $\rho = 10^{14} m^{-2}$



Diszlokációk térelmélete

$$\partial_i u_j := \beta_{ij} = \beta_{ij}^p + \beta_{ij}^e \qquad \frac{d\underline{u}}{d\underline{r}} := \hat{\beta} = \hat{\beta}^e + \hat{\beta}^p$$

Diszlokáció sűrűség tensor

$$\alpha_{ij} = \mathbf{e}_{ikl} \partial_k \beta_{lj}^p = -\mathbf{e}_{ikl} \partial_k \beta_{lj}^e \qquad \hat{\alpha} = \nabla \times \hat{\beta}^p = -\nabla \times \hat{\beta}^e$$

$$b_j = \int_A \alpha_{ij} dA_i$$

$$b_j = - \int_A \mathbf{e}_{ikl} \frac{\partial}{\partial r_k} \beta_{lj}^e dA_i = - \oint \beta_{ij}^e ds_i = - \oint du_j.$$

Egyedi diszlokáció

$$\alpha_{ij} = l_i b_j \delta(\xi) \quad (\hat{\alpha} = \underline{l} \circ \underline{b} \delta(\xi))$$

Plasztikus disztorzio

$$\beta_{ij}^p(\vec{r}) = n_i b_k \delta(\zeta)$$

Szorozzuk meg a rugalmas állandóval és vegyük a div-jét

$$\frac{\partial}{\partial r_i} C_{ijkl} \frac{\partial u_k}{\partial r_l} = \frac{\partial}{\partial r_i} C_{ijkl} \beta_{kl}^p = f_j \left(\text{div } \hat{C} \frac{d\underline{u}}{d\underline{r}} = \text{div } \hat{C} \hat{\beta}^p = \underline{f} \right)$$

Csavardiszlokáció elmozdulástere

$$u_3 = \frac{b}{2\pi} \varphi$$

Egyenes diszlokáció feszültségtere $\sigma \propto b/r$

$$\sigma_{11} = \frac{\mu b}{2\pi(1-\nu)} \frac{y(3x^2 + y^2)}{(x^2 + y^2)^2}$$

$$\sigma_{22} = \frac{\mu b}{2\pi(1-\nu)} \frac{y(x^2 - y^2)}{(x^2 + y^2)^2}$$

$$\sigma_{12} = \frac{\mu b}{2\pi(1-\nu)} \frac{x(x^2 - y^2)}{(x^2 + y^2)^2}$$

$$\sigma \propto b/r \text{ így } E \propto b^2$$

$$2 \left(\frac{b}{2} \right)^2 < b^2$$

$$\underline{b} = \underline{a}_i$$

$$\text{Fcc } \underline{b} = \frac{1}{2}(1, 1, 0)$$

$$\text{Bcc } \underline{b} = \frac{1}{2}(1, 1, 1)$$

Diszlokációk kölcsönhatása

Rugalmas energia

$$E = \frac{1}{2} \int \sigma_{ij} \partial_i u_j dV$$

Legyen

$$\sigma_{ij} = \sigma_{ij}^{\text{ext}} + \sigma_{ij}^{\text{disl}}$$

$$u_i = u_i^{\text{ext}} + u_i^{\text{disl}}$$

Ekkor

$$E = \frac{1}{2} \int \left[\sigma_{ij}^{\text{ext}} \partial_i u_j^{\text{ext}} + \sigma_{ij}^{\text{ext}} \partial_i u_j^{\text{disl}} + \sigma_{ij}^{\text{disl}} \partial_i u_j^{\text{ext}} + \sigma_{ij}^{\text{disl}} \partial_i u_j^{\text{disl}} \right] dV$$

Mivel

$$\sigma_{ij} = C_{ijkl} \partial_k u_l$$

$$E = E_{\text{ext}} + E_{\text{disl}} + \int \sigma_{ij}^{\text{ext}} \partial_i u_j^{\text{disl}} dV$$



$$E_{int} = \int \sigma_{ij}^{ext} \partial_i u_j^{disl} dV$$

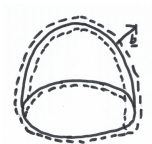
Parciális integrálással

$$E_{int} = \int \partial_i (\sigma_{ij}^{ext} u_j^{disl}) dV - \int (\partial_i \sigma_{ij}^{ext}) u_j^{disl} dV$$

Mivel

$$\partial_i \sigma_{ij}^{ext} = 0$$

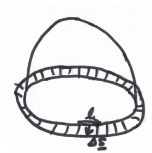
$$E_{int} = \int \partial_i (\sigma_{ij}^{ext} u_j^{disl}) dV = \int (\sigma_{ij}^{ext} u_j^{disl}) dn_i$$



\underline{u} -nak \underline{b} ugrása van a vágási felületen!

$$E_{int} = b_j \int_f \sigma_{ij}^{ext} dn_i$$

Ha a vonal megmozdul az vonal által húzott övre



$$\Delta E_{int} = b_j \oint_v \sigma_{ij}^{ext} (\Delta \underline{r} \times \underline{dl})_i$$



$$\Delta E_{int} = \oint_v (d\mathbf{l} \times (\hat{\sigma}^{ext} \mathbf{b}))_i \Delta r_i$$

Peack Koehler erő

$$\underline{f} = (\hat{\sigma}^{ext} \mathbf{b}) \times \underline{l}$$

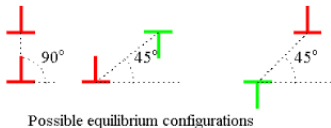
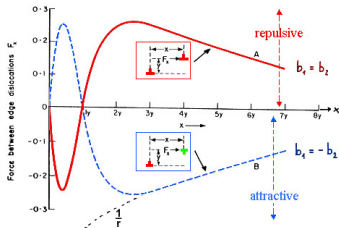
Diszlokációk kölcsönhatása

Kúszásnál csak a csúszósíkra eső vetület számít

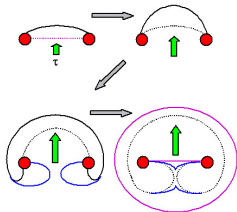
$$f_{\text{glide}} = [(\hat{\sigma}^{\text{ext}} \underline{b}) \times \underline{l}] \frac{b}{b} = \underline{n} \hat{\sigma}^{\text{ext}} \underline{b} = \tau b \quad \text{ahol} \quad \underline{n} = \frac{\underline{b}}{b} \times \underline{l}$$

Éldiszlokációk

$$f = b_1 b_2 \frac{\mu}{2\pi(1-\nu)} \frac{x(x^2 - d^2)}{(x^2 + d^2)^2}$$



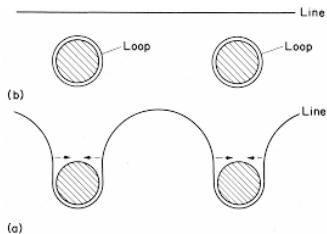
Frank-Read forrás



Folyáshatár (Taylor reláció)

$$\tau_f = \alpha \mu b \sqrt{\rho}$$

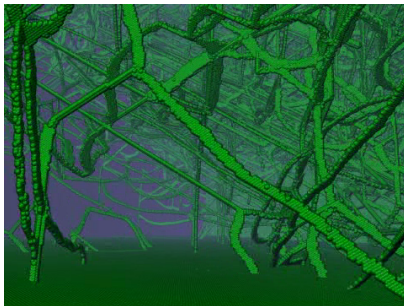
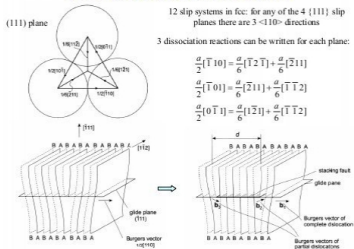
Vonalfeszültség $\tau_l = E_l/R$

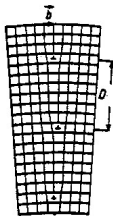


Kritikus feszültség

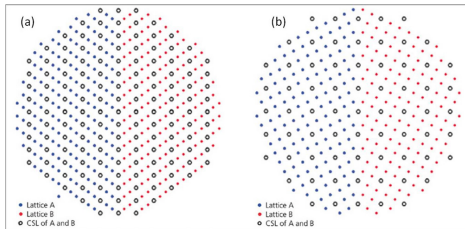
$$\tau_c = \alpha \frac{1}{l}$$

Shockley partial dislocations in fcc crystals

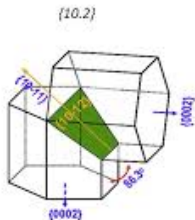
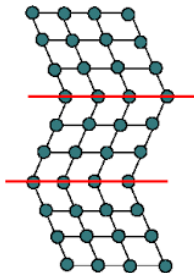




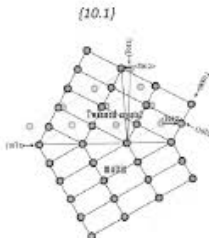
Kisszögű szemcsehatár



Koincidencia rács (Σn)



L. Wu, A. Jain, D.W. Brown, G.M. Steira,
S.R. Agnew, B. Clausen, D.E. Fielden,
P.K. Liaw. *Acta Materialia* 56 (2008) 688–695



I. Kim, W.S. Joang, J. Kim, K.T. Park,
D.H. Shin *Scripta Materialia* 45 (2001)
575–581