



# *Kondenzált anyagok fizikája*

## *Mágnesség*

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Lagrange függvény

$$\mathcal{L} = \frac{1}{2}mv^2 + qv_i A_i(\underline{r}(t), t) - q\phi(\underline{r}(t))$$

Mozgásegyenlet

$$\frac{d}{dt} \left( \frac{\partial \mathcal{L}}{\partial v_j} \right) - \frac{\partial \mathcal{L}}{\partial r_j} = 0$$

Így

$$\frac{d}{dt} [mv_j + qA_j(\underline{r}(t), t)] - qv_i \frac{\partial A_i}{\partial r_j} + q \frac{\partial \phi}{\partial r_j} = 0$$

$$ma_j + q \frac{\partial A_j}{\partial t} + qv_i \frac{\partial A_j}{\partial r_i} - qv_i \frac{\partial A_i}{\partial r_j} + q \frac{\partial \phi}{\partial r_j} = 0$$

## Elektron mágneses térben

Lorentz erő

$$m\mathbf{a}_j + q \left( \dot{\mathbf{A}}_j + \frac{\partial \phi}{\partial \mathbf{r}_j} \right) + q\mathbf{v}_j \left( \frac{\partial \mathbf{A}_j}{\partial \mathbf{r}_i} - \frac{\partial \mathbf{A}_i}{\partial \mathbf{r}_j} \right) = 0$$
$$-\underline{E} \quad - \times \underline{B}$$

Kanonikus impulzus

$$\mathbf{p}_j = \frac{\partial \mathcal{L}}{\partial \mathbf{v}_j} = m\mathbf{v}_j + q\mathbf{A}_j$$

Hamilton függvény

$$H = \mathbf{v}_i \mathbf{p}_i - \mathcal{L} = \frac{1}{2m} (\underline{\mathbf{p}} - q\underline{\mathbf{A}})^2 + q\phi$$

Elektronra

$$H = \frac{1}{2m_e} (\underline{\mathbf{p}} + e\underline{\mathbf{A}})^2 - e\phi$$

# Einstein-de Haas kísérlet

Köráramra

$$M = IA = I r^2 \pi = env r^2 \pi$$

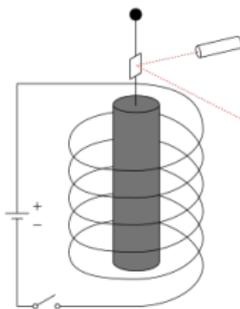
$$N = m v r 2 \pi r q n$$

$$\frac{M}{N} = \frac{e}{2m}$$

Owen Willans Richardson 1908

Einstein és de Haas 1915

John Quincy Stewart 1918



$$\frac{M}{N} = \frac{e}{2m} g'$$



Homogén mágneses térben

$$\underline{A} = \frac{1}{2} \underline{B} \times \underline{r}$$

ekkor

$$\underline{B} = \text{rot} \underline{A}, \quad \text{div} \underline{A} = 0$$

Ekkor  $H$

$$H = \frac{1}{2m_e} p^2 + \frac{e}{2m_e} \underline{p}(\underline{B} \times \underline{r}) + \frac{e^2}{8m_e} (\underline{B} \times \underline{r})^2$$

Átírható

$$H = \frac{1}{2m_e} p^2 + \frac{e}{2m_e} \underline{B}(\underline{r} \times \underline{p}) + \frac{e^2}{8m_e} (\underline{B} \times \underline{r})^2$$

Felhasználva, hogy  $\underline{L} = \underline{r} \times \underline{p} = \hbar \underline{l}$



$$H = \frac{1}{2m_e} p^2 + \mu_B \underline{l} \underline{B} + \frac{e^2}{8m_e} \vec{B}^2 r_{\perp}^2$$

ahol

$$\mu_B = \frac{\hbar e}{2m_e}$$

a Bohr magneton

Ha van spin is

$$H = \frac{1}{2m_e} p^2 + \mu_B (\underline{l} + g_e \underline{s}) \underline{B} + \frac{e^2}{8m_e} \vec{B}^2 r_{\perp}^2$$

ahol  $g_e \approx 2$

Sok elektronra

$$H = \sum_i \left[ \frac{1}{2m_e} p_i^2 + \mu_B (\underline{l}_i + g_e \underline{s}_i) \underline{B} + \frac{e^2}{8m_e} \vec{B}^2 r_{\perp i}^2 \right]$$



$$M = \frac{1}{V} \frac{\sum_i m_i e^{-\frac{E_i(B)}{k_B T}}}{\sum_i e^{-\frac{E_i(B)}{k_B T}}}$$

Bevezetve

$$F = -k_B T \ln \left( \sum_i e^{-\frac{E_i(B)}{k_B T}} \right)$$

Mivel egy dipól energiája

$$E_i = -m_i B$$

$$\frac{dF}{dB} = \frac{k_B T}{k_B T} \frac{\sum_i \frac{dE_i}{dB} e^{-\frac{E_i(B)}{k_B T}}}{\sum_i e^{-\frac{E_i(B)}{k_B T}}} = -VM$$

$$M = -\frac{1}{V} \frac{dF}{dB}$$

Szuszceptibilitás

$$\chi' = \left. \frac{dM}{dB} \right|_{B=0} \quad \left( \chi = \frac{dM}{dH}, \quad \chi \approx \mu_0 \chi' \right)$$



Egy elektront tekintve

$$F_1 = -k_B T \ln \left( \sum_i e^{-\frac{e^2}{8m_e} \frac{B^2 r_{\perp}^2}{k_B T} i} \right)$$

Kis tér esetén csak az alapállapotot tekintve

$$F_1 \cong \frac{e^2}{8m_e} B^2 \langle r_{\perp}^2 \rangle$$

$N$  elektronra

$$F \cong N \frac{e^2}{8m_e} B^2 \langle r_{\perp}^2 \rangle$$

Innen

$$\chi' = -\frac{e^2}{4m_e} \frac{N \langle r_{\perp}^2 \rangle}{V}$$

Mivel gömbszimmetrikus esetre  $\langle r_{\perp}^2 \rangle = \frac{2}{3} \langle r^2 \rangle$

Mivel gömbszimmetrikus esetre

$$\langle r_{\perp}^2 \rangle = \frac{2}{3} \langle r^2 \rangle$$

$$\chi' = -n \frac{e^2}{6m_e} \langle r^2 \rangle$$

ahol  $\langle r^2 \rangle \approx a_0^2$

Így

$$\chi = -n \frac{e^2 \mu_0}{6m_e} a_0^2$$



Klasszikus határeset, "valahonnan van egy mágneses momentum"  $\mu$

$$Z = \int e^{\frac{\mu B}{k_B T}} d\Omega = 2\pi \int_0^\pi e^{\frac{\mu B}{k_B T} \cos \Theta} \sin \Theta d\Theta$$

$$Z = 2\pi \int_{-1}^1 e^{\frac{\mu B}{k_B T} x} dx = 4\pi \frac{k_B T}{\mu B} \frac{e^{\frac{\mu B}{k_B T}} - e^{-\frac{\mu B}{k_B T}}}{2}$$

$$Z = 4\pi \frac{k_B T}{\mu B} \operatorname{sh} \left( \frac{\mu B}{k_B T} \right)$$

Innen

$$F_1 = -k_B T \ln \left[ 4\pi \frac{k_B T}{\mu B} \operatorname{sh} \left( \frac{\mu B}{k_B T} \right) \right]$$

$$\operatorname{sh}(x) = \frac{e^x - e^{-x}}{2} \approx x + \frac{x^3}{6}$$

$$\ln \left( \frac{\text{sh}(x)}{x} \right) \approx \frac{x^2}{6}$$

$$F = -N \frac{\mu^2 B^2}{6k_B T}$$

$$\chi = \mu_0 n \frac{\mu^2}{6k_B T}$$

Langevin függvény

$$L(x) = \frac{d}{dx} \ln \left( \frac{\text{sh}(x)}{x} \right) = -\frac{1}{x} + \frac{\text{ch}(x)}{\text{sh}(x)}$$

$$\langle \mu \rangle = \mu L \left( \frac{\mu B}{k_b T} \right)$$

Kvantumos esetre  
Adott  $J$ -re

$$\mu_B(l + g_e S) = \mu_B g_J J$$

Így

$$H = \mu_B g_J J_z B, \quad J_z = -J \dots J$$

$2J + 1$  eset

$$Z = \sum_{J_z=-J}^J e^{-\beta E_0} e^{-\beta g_J \mu_B J_z B}$$

$$Z = e^{-\beta E_0} e^{\beta g_J \mu_B J B} \frac{e^{-\beta g_J \mu_B (2J+1) B} - 1}{e^{-\beta g_J \mu_B B} - 1}$$



$$Z = e^{-\beta E_0} \frac{e^{\beta g_J \mu_B (J + \frac{1}{2}) B} - e^{-\beta g_J \mu_B (J + \frac{1}{2}) B}}{e^{\beta g_J \mu_B \frac{B}{2}} - e^{-\beta g_J \mu_B \frac{B}{2}}}$$

$$Z = e^{-\beta E_0} \frac{\text{sh}(\beta g_J \mu_B (J + \frac{1}{2}) B)}{\text{sh}(\beta g_J \mu_B \frac{B}{2})}$$

Mágneszettség

$$M = -\frac{1}{V} \frac{dF}{dB} = n g_J \mu_B J B_J(\beta g_J \mu_B J B)$$

Brillouin függvény

$$B_J(x) = \frac{2J+1}{2J} \text{cth}\left(\frac{2J+1}{2J}x\right) - \frac{1}{2J} \text{cth}\left(\frac{1}{2J}x\right)$$

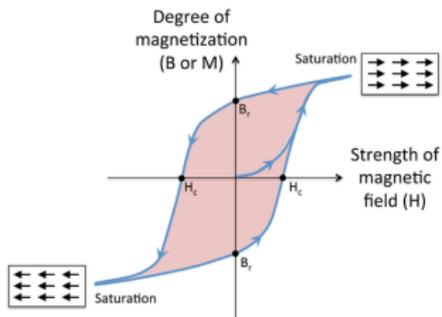
$$B_{\frac{1}{2}}(x) = \text{th}(x)$$



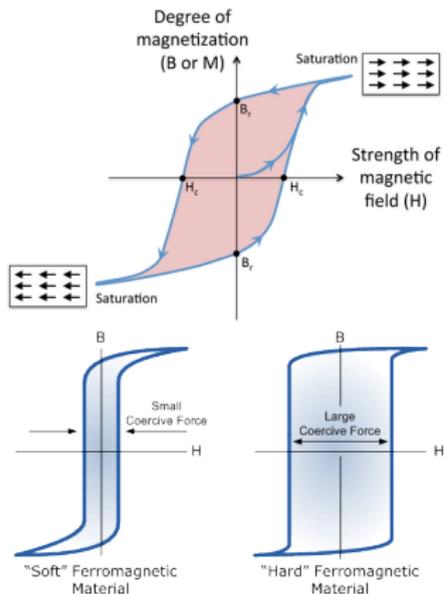
$$B_J(x) \approx \frac{J+1}{J} \frac{x}{3}$$

$$\chi = n\mu_0 \frac{g_J^2 \mu_B^2 J(J+1)}{3k_B T}$$

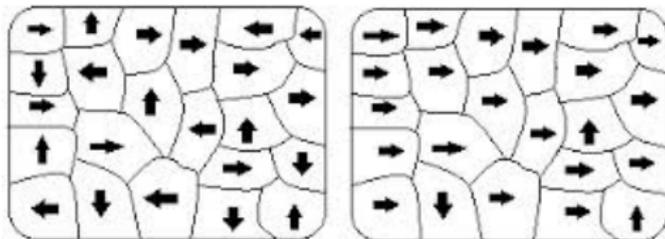
## Hiszterézis



## Hiszterézis



Domén szerkezet

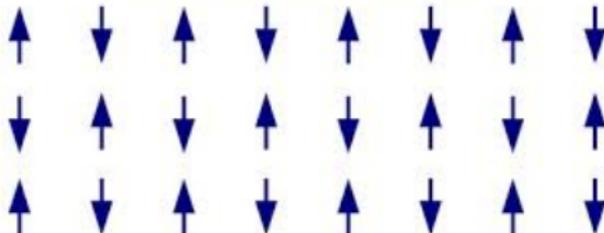


Unmagnetized

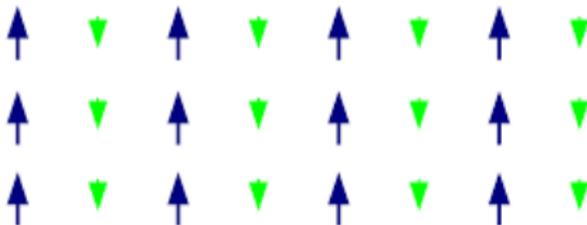
Magnetized

Antiferromágnesség

### Antiferromagnetism in Physics



## Ferrimágnesség



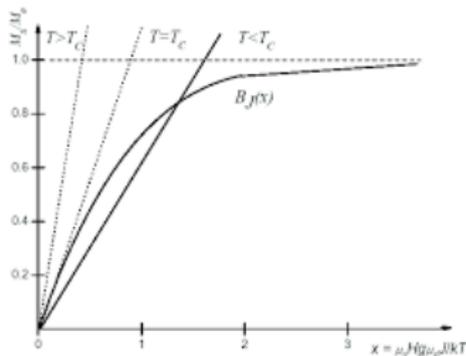
$$M = n g_J \mu_B J B_J \left( \frac{g_J \mu_B J}{k_B T} B \right)$$

Átlagtér elmélet

$$B_{\text{eff}} = \mu_0 H + \lambda M$$

Ha  $H = 0$

$$M = n g_J \mu_B J B_J \left( \lambda \frac{g_J \mu_B J}{k_B T} M \right)$$



$$\left. \frac{dB_J}{dx} \right|_{x=0} = \frac{J+1}{3J}$$

Kritikus pont

$$ng_J^2 \mu_B^2 J^2 \lambda \frac{J+1}{3J} \frac{1}{k_B T_c} = 1$$



$T_c$  fölött a paramágneses  $\chi$

$$M = n g_J \mu_B J \left[ \frac{g_J \mu_B J}{k_B T} (\mu_0 H + \lambda M) \right] \frac{J+1}{3J}$$

$$M = n \frac{g_J^2 \mu_B^2 J(J+1)}{3} \frac{\mu_0}{k_B T} H + \frac{k_B T_c}{k_B T} M$$

Innen

$$M = n \frac{g_J^2 \mu_B^2 J(J+1)}{3(T - T_c)} \mu_0 H$$

Szuszceptibilitás  $T_c$  fölött

$$\chi = n \frac{g_J^2 \mu_B^2 J(J+1)}{3(T - T_c)} \mu_0$$

Mágnesezettség  $T_c$  alatt. A

$$M = n g_J \mu_B J B_J \left( \lambda \frac{g_J \mu_B J}{k_B T} M \right)$$

egyenlet megoldása sorfejtéssel

$$M = n \frac{g_J^2 \mu_B^2 J(J+1)}{3} \frac{\mu_0}{k_B T} H + \frac{k_B T_c}{k_B T} M + AM^3$$

ha  $H = 0$ .

$$M = \frac{k_B T_c}{k_B T} M + AM^3$$

Innen

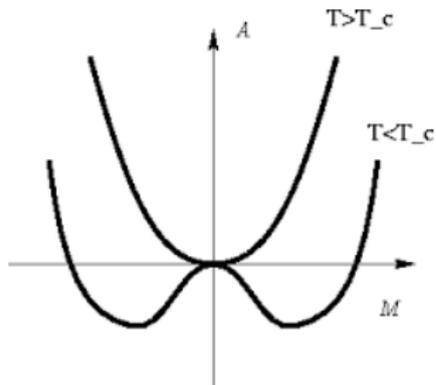
$$M \propto (T_c - T)^{\frac{1}{2}}$$

Ha  $T = T_c$

$$M \propto H^{\frac{1}{3}}$$

Fenomenológikus  $F(M)$  reláció ami  $M \leftrightarrow -M$  invariáns

$$\frac{F}{V} = f_0 + A(T)M^2 + \frac{B(T)}{2}M^4$$



$$A(T) = a(T - T_c), \quad B = \text{const} > 0$$

Egyensúly

$$\frac{dF}{dM} = 0, \quad 2A(T)M + 2BM^3 = 0$$

Innen a minimumok

$$M = \pm \sqrt{-\frac{A(T)}{B}}$$

Így

$$M \propto \sqrt{T_c - T}$$

Ha van mágneses tér

$$\frac{F}{V} = f_0 + A(T)M^2 + \frac{B}{2}M^4 - \mu_0 MH$$

$$\frac{dF}{dM} = 0, \quad 2A(T)M + 2BM^3 - \mu_0 H = 0$$

Ha  $T > T_c$

$$M = \frac{\mu_0 H}{2a(T - T_c)}$$

Így

$$\chi = \frac{\mu_0}{2a(T - T_c)}$$

$T_c$  alatt

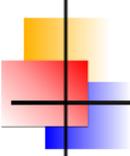
$$2A(T)M + 2BM^3 = \mu_0 H$$

Deriválva

$$2A(T) \frac{dM}{dH} + 6BM^2 \frac{dM}{dH} = \mu_0$$

Mivel  $M^2 = -A(T)/B$

$$\chi_m = -\frac{\mu_0}{4A(T)} = \frac{\mu_0}{4a(T_c - T)}$$



# Landau elmélet

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Ha  $T = T_c$

$$M \propto H^{\frac{1}{3}}$$

$$\begin{array}{l|l} C(T) \propto |T - T_c|^{-\alpha} & \alpha = 0 \\ \chi(T) \propto |T - T_c|^{-\gamma} & \gamma = 1 \\ M(T) \propto |T_c - T|^\beta & \beta = \frac{1}{2} \\ M(H) \propto H^{1/\delta} & \delta = 3 \end{array}$$

Skálatörvények

$$\alpha + 2\beta + \gamma = 2$$

$$\gamma = \beta(\delta - 1)$$

Szabadenergia skálázás

$$F(\lambda^{a_1} t, \lambda^{a_2} H) = \lambda F(t, H), \quad t = \frac{T - T_c}{T_c}$$



A doménfal extra energiájának figyelembe vétele

$$F = F_0 + \int \left[ A(T)M^2 + \frac{B(T)}{2}M^4 + l^2 a(\nabla M)^2 \right] dV$$

Funkcionális derivált:  $F\{M\}$  egy funkcionál

$$\delta F = \int \left[ \frac{\delta F}{\delta M} \delta M \right] dV$$

Egyesúlyban

$$\frac{\delta F}{\delta M} = 0$$

Egyensúlyhoz közel

$$\frac{\partial M}{\partial t} = -D \frac{\delta F}{\delta M}$$

Hamilton operátor

$$H\{s\} = -J \sum_{\langle i,j \rangle} s_i s_j + h \sum_i s_i$$

ahol  $\langle i,j \rangle$  első szomszédra történő összegezést jelent.  
2D-ben van analitikus megoldás!

