Kondenzált anyagok fizikája

Szupravezetés

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ELTE

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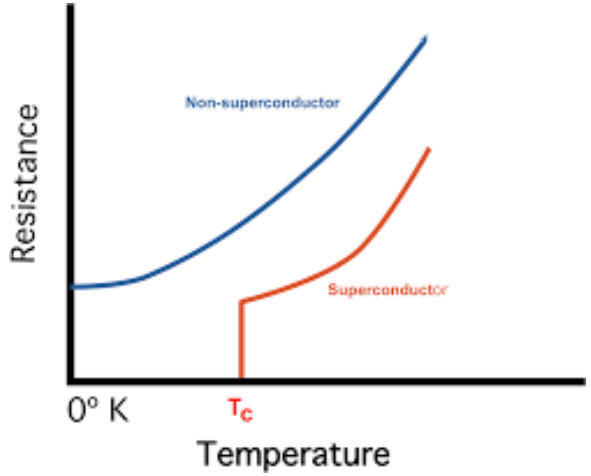
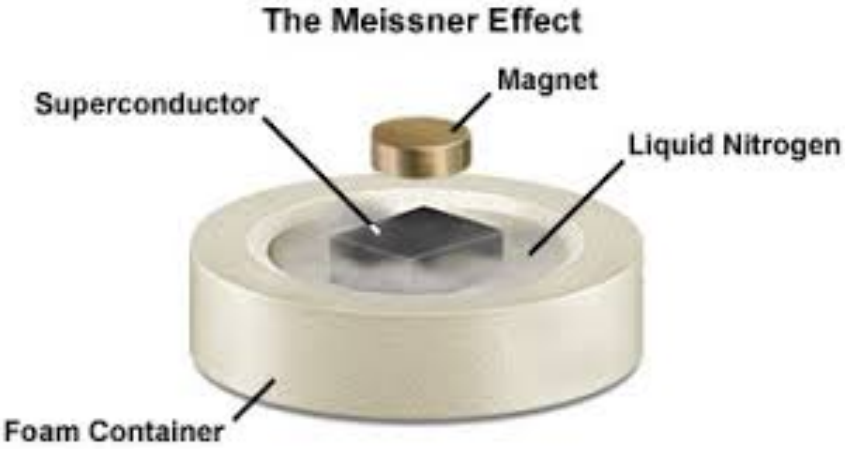
Alapjelenségek

Heike Kammerligh Oness, 1911 Leiden



Meissner effektus

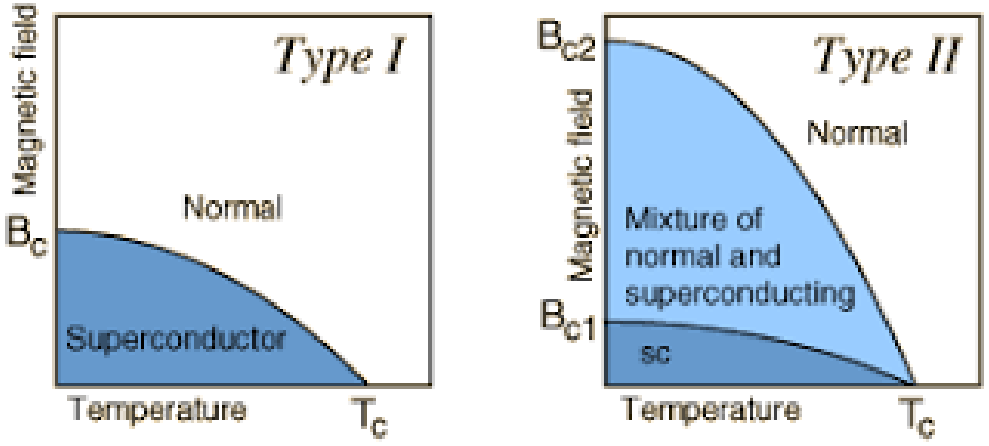


Ideális diamágnes

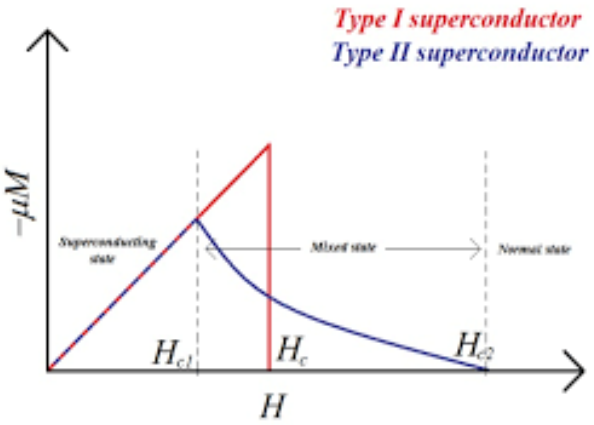
$B = 0$

$$B = \mu_0(H + M) = 0 \quad \Rightarrow M = -H$$

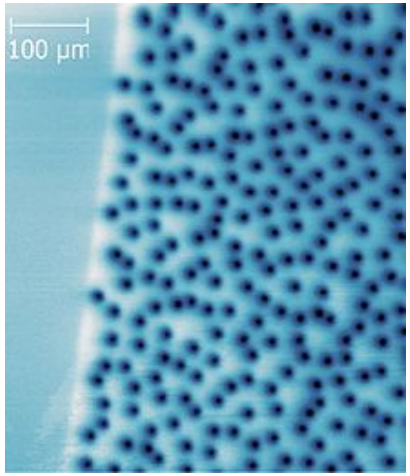
Ideális diamágnes.
Fázisdiagramm

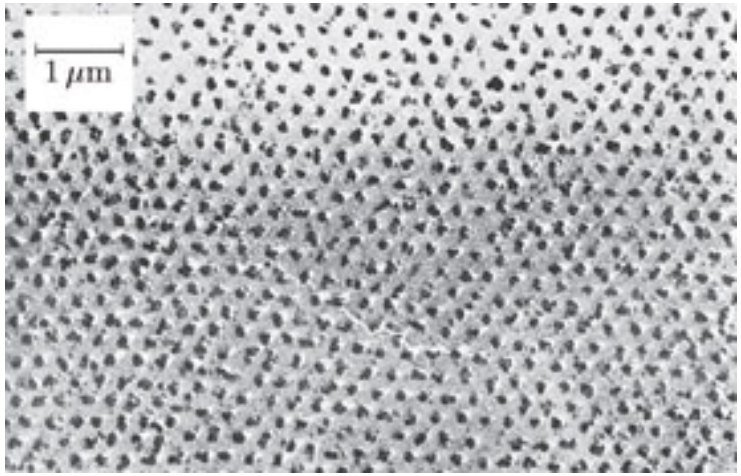


II fajú szupravezető



Szupravezető vortex $\phi = h/2e$





Szupravezető állapotegyenlete

Mi van a differenciál Ohm törvény helyett?

$$\underline{j} = -en\underline{v}$$

Mozgásegyenlet

$$m\dot{\underline{v}} = -e\underline{E}$$

Innen

$$\dot{\underline{j}} = \frac{ne^2}{m}\underline{E}$$

Indukciós törvény

$$rot\underline{E} = -\dot{\underline{B}}$$

Innen

$$\frac{d}{dt}\left(\frac{m}{ne^2}rot\underline{j}\right) = -\frac{d}{dt}\underline{B}$$

Szupravezető állapotegyenlete

Innen

$$rot\underline{j} = -\frac{ne^2}{m}\underline{B}$$

$$\underline{j} = -\frac{ne^2}{m}\underline{A}, \quad div\underline{j} = 0 \quad \rightarrow \quad div\underline{A} = 0$$

Bevezetve

$$\lambda^2 = \frac{m}{ne^2\mu_0}$$

$$\underline{B} = -\mu_0\lambda^2 rot\underline{j}$$

Felhasználva, hogy

$$rot\underline{B} = \mu_0\underline{j} \quad (\dot{\underline{D}} \approx 0)$$

$$rotrot\underline{B} = \mu_0 rot\underline{j} = -\frac{1}{\lambda^2}\underline{B}$$

London egyenlet

$$rotrot = graddiv - \triangle$$

$$\triangle \underline{B} = \frac{1}{\lambda^2} \underline{B}$$

Szupravezető-normál határréteg

$$\frac{d^2 B_z}{dx^2} = \frac{1}{\lambda^2} B_z$$

$$B_z(x) = \begin{cases} B_0 & x < 0 \\ B_0 e^{-\frac{x}{\lambda}} & x > 0 \end{cases}$$

Vortex tere

Hengerszimmetrikus probléma.

$$\frac{\lambda^2}{r}\frac{d}{dr}\left(r\frac{d}{dr}\right)B_z(r) = B_z(r)$$

Fluxus kvantálás

$$2\pi\int_0^\infty rB_z(r)dr = \phi_0$$

Azaz

$$-2\pi\lambda^2\left(r\frac{d}{dr}\right)B_z(r)\Big|_{r=0} = \phi_0$$

Megoldás

$$B_z(r) = \frac{\phi_0}{2\pi\lambda^2}K_0\left(\frac{r}{\lambda}\right)$$

ahol $K_0(x)$ a nulladrendű módosított Bessel függvény amely

$$K_0(x) \approx \ln(2x) \quad x << 1$$

Fluxus kvantálás

Sommerfeld

$$\oint \underline{p} d\underline{s} = nh$$

$$\underline{p} = m\underline{v} - e\underline{A}$$

$$m \oint \underline{v} d\underline{s} - e \oint \underline{A} d\underline{s} = nh$$

$$\frac{m}{en} \oint \underline{j} d\underline{s} - e \oint \underline{A} d\underline{s} = nh$$

Legyen $\underline{j} = 0$

$$-e \oint \underline{A} d\underline{s} = -e \int rot\underline{A} d\underline{f} = -e\phi_0 = nh$$

Innen $\phi_0 = h/e$ de $\phi_0 = h/2e$, Cooper pár

Ginzburg Landau elmélet

$$F = F_0 + \int \left[a(T)|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{1}{2m}|(-i\hbar \bigtriangledown -2e\underline{A})\psi|^2 + \frac{B^2}{2\mu} \right] dV$$

ahol $a(T) = a_0(T - T_c)$
Egyensúly

$$\frac{\delta F}{\delta \psi} = 0 \qquad \frac{\delta F}{\delta \underline{B}} = 0$$

$$a(T)\psi + \beta|\psi|^2\psi + \frac{1}{2m}(-i\hbar \bigtriangledown -2e\underline{A})^2\psi = 0$$

$$\bigtriangledown \times \underline{B} = \mu_0 \underline{j}; \qquad \underline{j} = \frac{2e}{m} Re\{\psi^*(-i\hbar \bigtriangledown -2e\underline{A})\psi\}$$