

Advanced Laboratory Course, Part I

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Report on

E-103:
Franck-Hertz Experiment

Handed in by

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1 Introduction and Theoretical Foundations

1.1 Idea of the Experiment

NIELS BOHR's model of the atom consisting of several discrete energy levels was an attempt to explain the observed line spectra of the elements, in particular the hydrogen spectrum. In 1913, JAMES FRANCK and GUSTAV HERTZ carried out an experiment which demonstrated the existence of those energy levels without relying on the methods of optical spectroscopy. The result of the so-called "Franck-Hertz experiment" indicates that atoms can absorb energy only in discrete quanta, regardless of the way the energy is transferred to the atom (e.g. by optical excitation or mechanical collisions).

In our experiment we will use a setup similar to that of FRANCK and HERTZ to inspect the quantized absorption of energy by mercury atoms. The aim of the experiment is to determine the mean excitation energy of the first three excited states of mercury. In addition, we will observe resonance light emitted from the tube by means of a photomultiplier. Furthermore, we shall examine the influence of important parameters like temperature and back voltage.

1.2 Optical, Thermal and Mechanical Excitation of Atoms

According to BOHR's model, the energy carried away by a photon which is emitted whenever an electron undergoes a transition from a certain state i to a less energetic state j is given by

$$h\nu = E_i - E_j,$$

where E_i and E_j ($E_i > E_j$) are the energies associated with the two levels. The reverse should also be possible: by supplying energy of the amount $h\nu$ to the atom, it can be brought from the level E_j to E_i . This process is called *excitation*.

There are many different methods of exciting atoms. For example, light of a frequency corresponding exactly to the energy difference between the two states can be used to excite the atom by absorption of a photon (*induced absorption*). In the transition which follows the excitation, $E_i \rightarrow E_j$, the photon is reemitted in a direction independent from the one of the incident photon (*spontaneous emission*).

Another method makes use of high temperature gases. Inelastic collisions between atoms of high kinetic energy can result in excitation. The ratio of excited atoms can be calculated according to

$$\frac{N_i}{N_0} = \exp\left(-\frac{E_i - E_0}{kT}\right),$$

where N_i , N_0 are the numbers of excited atoms or atoms in the ground state, T is the temperature of the gas and k is Boltzmann's constant.

However, the most wide-spread method of excitation is based on electron impact in discharges of low-pressure gases. Here, electrons and ions are accelerated in an electric field. In inelastic collisions with atoms, they transfer energy to their collision partners. This way, kinetic energy is converted into excitation energy.

No matter in what way the excitation was achieved, a common feature of all excited states is that they "decay" within a very short time, i.e. they lose their excitation energy in form of radiation. The decay time is determined by the life time of the state, which is usually of the order of $\approx 10^{-8}$ sec.

1.3 Collisions and Cross Sections

Collisions play an important role in the Franck-Hertz experiment. More specifically, we use mercury atoms colliding with electrons. Of course, collisions between two mercury atoms can also occur. We have to distinguish between two kinds of collisions:

- **elastic collisions:** the total amount of kinetic energy is constant. Since the transfer of kinetic energy during a collision depends on the masses of the involved particles, the electron hardly loses any energy when colliding with the much heavier mercury atom (consider $m_e/m_{Hg} \approx 2.6 \cdot 10^{-6}$). Its velocity remains roughly the same, only its direction may be changed.
- **inelastic collisions:** a certain amount of the kinetic energy is converted into some other type of energy, e.g. it may be used to excite a mercury atom. The electron can be slowed down considerably, apart from changing its direction. This is the type of collision which is relevant for the experiment.

The rate of collisions depends on a set of parameters, such as temperature (T) and pressure (p) of the gas. The mean free path for an electron (i.e. the mean distance between two collisions) is given by¹

$$\ell = \frac{kT}{\sigma \cdot p}, \quad (1)$$

where σ denotes the total cross section:

$$\sigma = \frac{\text{\# of collisions per unit time}}{(\text{\# of incoming particles per unit time}) \times (\text{\# of collision partners per unit area})}.$$

The kinetic theory of gases states that the *geometrical cross section* can be calculated from the assumed gaskinetic diameters d_1, d_2 of the two colliding particles:

$$\sigma_{geo} = \pi \cdot \left(\frac{d_1}{2} + \frac{d_2}{2} \right)^2. \quad (2)$$

Considering a) two particles of equal mass and b) a "heavier" particle and a very light one, we get

$$\sigma_{(Hg,Hg)} = \pi \cdot d_{Hg}^2 \quad \text{resp.} \quad \sigma_{(e^-,Hg)} = \pi \cdot (d_{Hg}/2)^2. \quad (3)$$

In general, the cross section for excitation, σ_{exc} , depends on the energy of the incoming particles (assuming that the collision partners are at rest.) This dependency is shown in fig. 1 for mercury and hydrogen. Simultaneously, another process can take place. Above a certain critical energy, the electrons can ionize the mercury atoms. Thus, we don't get excited states any more, but rather electron – ion pairs. (If the incoming electron has more energy than the amount needed for ionization, the rest of course may be used to excite the ion.) The problem with ionization in our experiment is that it can destroy the Franck-Hertz tube, since ionized gas is a conductor. That means, as soon as there are ions present in the mercury vapour, the current will increase considerably.

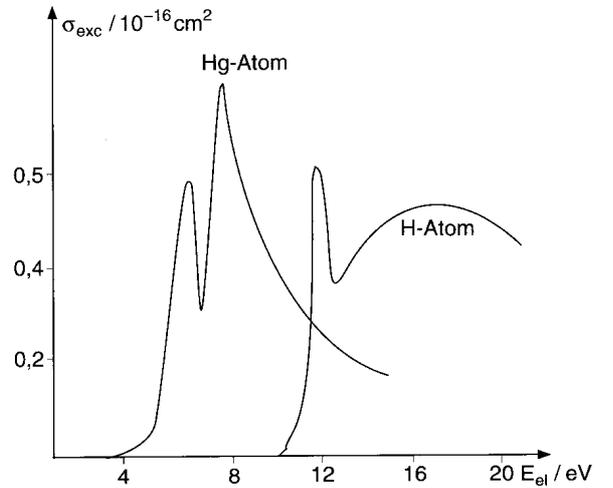


Figure 1: Excitation cross sections for ($n = 1 \rightarrow n = 2$) in H and for ($6^1S_0 \rightarrow 6^3P_1$) in Hg

1.4 Some Details About the Experiment

A tube containing a cathode (K), an anode (A) and a third, grid-like electrode (G) is filled with mercury vapour of a low pressure (usually about 10^{-2} mbar). Figure 2 shows the arrangement of the components. Between K and G, the electrons emitted from the cathode are accelerated by a voltage U ; they thus receive the energy $e \cdot U$. The anode is kept at a potential $U_A = U - \Delta U$, so that the electrons are decelerated after passing the grid G. Only if their energy is larger than $e \cdot \Delta U$, the electrons can reach the anode and form a current signal.

Fig. 3 displays the measurement of the current of electrons reaching A as a function of the accelerating voltage U . Between $U = 0$ V and 4.9 V the current increases, according to the characteristic curve of a diode. Above $U = 5$ V, the current decreases, reaches a minimum value and increases again until $U = 9.8$ V (and so on). How can we explain the periodic form of this curve?

The reason for this behaviour of the current is that the electrons undergo inelastic collisions with the Hg atoms, resulting in excitation of the latter:



¹assuming that the mercury vapour can approximatively be described by the ideal gas equation $p \cdot V = N \cdot k \cdot T$, and using $n = N/V$ to rewrite $\ell = \frac{1}{\sigma \cdot n}$

where $\Delta E_{kin} \approx E_{exc}$. Because of their loss of kinetic energy, some electrons are no longer able to reach the anode; therefore, the current drops. Others still have enough energy to surpass the back voltage; they contribute to the current which never goes back to zero. It is interesting to see that both the minimum and maximum values of the current, I_{min} and I_{max} , grow larger with increasing accelerating voltage – just like the expected behaviour of a diode. The fact that there are several maxima / minima in the diagram can be explained by taking into account those electrons that

- either have enough initial kinetic energy to excite more than one mercury atom (thus losing the same amount of energy twice, i.e. $\Delta E_{kin} = 2 \cdot 4.9 \text{ eV}$)
- or gain enough energy in the electric field between two collisions to excite several atoms.

The second item is more important, provided that the accelerating voltage U is sufficiently high. It is possible to find out the amount of energy transferred from the e^- to the Hg atoms in each collision by measuring the distance between two characteristic points in the current curve, e.g. maxima or minima.²

Elastic collisions can be neglected if the pressure of the vapour is sufficiently low, since here the electrons can lose only $\Delta E_{kin} = 4 \cdot \frac{m_e}{m_{Hg}}$, which is very small. In case there were no inelastic collisions between e^- and Hg atoms, the electron current would follow the dotted red curve in figure 3.

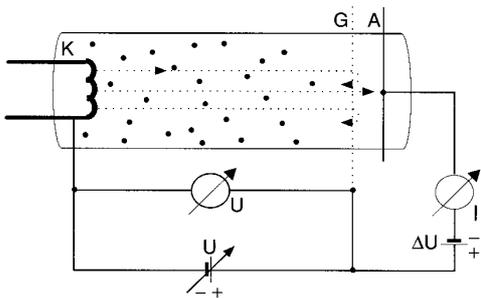


Figure 2: Sketch of a Franck-Hertz tube

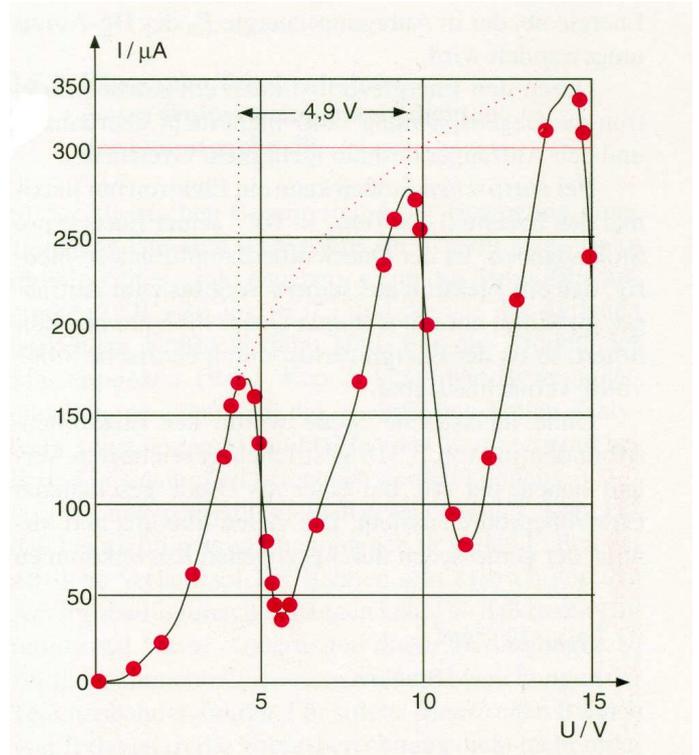


Figure 3: Typical measurement of the electron current

In conclusion, the most important results of the Franck-Hertz experiment are:

1. If the velocity of the electrons is smaller than some "critical" value, they only undergo elastic collisions with the mercury atoms. No excitation is observed.
2. If the velocity of an electron becomes critical, also inelastic collisions can take place. The electron then loses a certain *discrete* amount of energy which is transferred to the mercury atom.
3. The form of the curve in fig. 3 is determined by
 - (a) the energy dependence of σ_{exc} , cf. fig. 1, and
 - (b) the energetic distribution $N(E)$ of the electrons emitted from the hot cathode. (Because of the fact that the electrons are not monoenergetic, the original saw-blade profile of the Franck-Hertz curve is smoothed out.)

²See next section for further details on the excitation levels of mercury.

After a short time, the excited Hg atom goes back to its lowest energy state by emitting one or several photons of certain frequencies ν which are identical to the ones known to be "absorption frequencies" from spectroscopic observations:



This way, depending on which states are excited, one can observe the whole line spectrum of Hg in the emitted light. For the first excited state of mercury, the emitted radiation has the wavelength $\lambda = 253.7$ nm (ultraviolet light).

1.4.1 Properties of Mercury Vapour

The following table and figure contain some information on the element mercury and its energy levels.

atomic number	$Z = 80$
atomic mass	200.59 u
state at room temp.	liquid
gaskinetic diameter	$d_{Hg} = 0.44$ nm
conf. of ground state	$[\text{Xe}] 5d^{10}6s^2 \quad {}^1S_0$
conf. of excited states	$[\text{Xe}] 5d^{10}6sp \quad {}^3P_0$ (triplet) 3P_1 (triplet) 3P_2 $[\text{Xe}] 5d^{10}6sp \quad {}^1P_1$ $[\text{Xe}] 5d^{10}6s7s \quad {}^3S_1$ $[\text{Xe}] 5d^{10}6s7s \quad {}^1S_0$
life time of $6sp \quad {}^3P_1 \rightarrow 6s^2 \quad {}^1S_0$	117 ns

Table 1: Important features of mercury

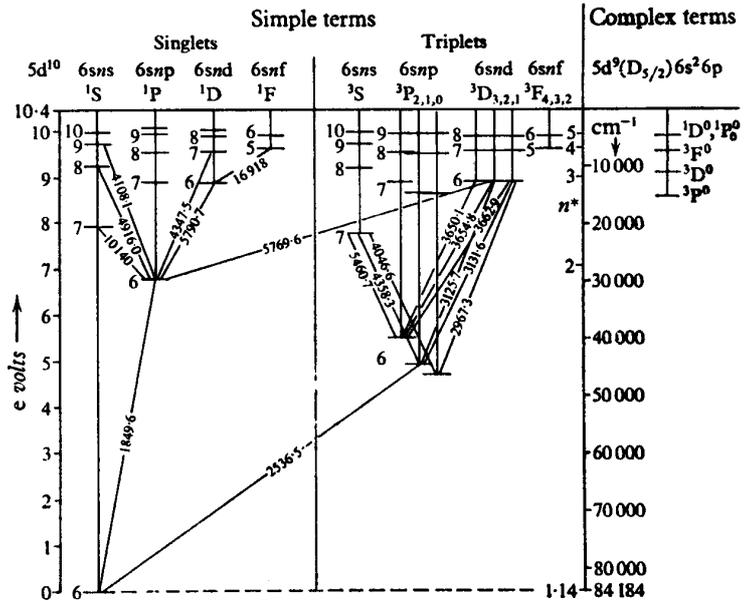


Figure 4: Term scheme of mercury

Interestingly, in most books the value of the excitation energy of mercury is referred to as 4.9 eV. Yet from the term scheme one sees that this value must be a weighted mean of various excitation energies for the different excited states, such as

$6sp \quad {}^3P_0$	$6sp \quad {}^3P_1$	$6sp \quad {}^3P_2$	$6sp \quad {}^1P_1$	$6s7s \quad {}^3S_1$	$6s7s \quad {}^1S_0$
4.67 eV	4.89 eV	5.46 eV	6.70 eV	7.73 eV	7.93 eV

Due to the small value of the mean free path for electrons in mercury vapour at high temperatures up to 200°C, the electrons will suffer frequent collisions with the atoms. It is unlikely that an electron can gather enough energy between two collisions to excite the higher states of mercury, so mainly the lower levels will be populated, in particular the 3P triplet states. Considering the different occupation probabilities for the triplet states 3P_0 , 3P_1 and 3P_2 , which are proportional to $(2J+1)$, we find the theoretical value of the excitation energy

$$\overline{E}_{exc} = \frac{1}{9} \cdot (1 \cdot 4.67 \text{ eV} + 3 \cdot 4.89 \text{ eV} + 5 \cdot 5.46 \text{ eV}) = 5.18 \text{ eV}. \quad (4)$$

Coupling of Angular Momenta and Selection Rules

For multi-electron atoms, two main mechanisms of angular momentum coupling are known:

- *LS-coupling* (also referred to as RUSSEL-SAUNDERS coupling), which represents the limit of light atoms, where the Coulomb repulsion between the electrons is stronger than spin-orbit interaction, and
- *jj-coupling*, which stands for the other extreme, namely heavy atoms with strong spin-orbit interaction.

In the first case, the total angular momentum J can be calculated according to $J = L + S$, where $S = \sum s_i$ and $L = \sum l_i$, whereas in the latter case, $J = \sum j_i = \sum (s_i + l_i)$. It is important to note that in most atoms (such as mercury with $Z = 80$), both mechanisms are present (*intermediate coupling*).

Within light atoms, transitions which intercombine a triplet state and a singlet state are strictly prohibited. The corresponding selection rule reads $\Delta S = 0$. However, this rule doesn't apply *strictly* for jj-coupling in heavier atoms. This is why transitions between $S = 1$ and $S = 0$ are possible, as shown in the term scheme, fig. 4. Other selection rules reflecting conservation of angular momentum and parity are $\Delta L = \pm 1$, $\Delta J = 0, \pm 1$ and $\Delta M_J = 0, \pm 1$ for LS-coupling, respectively $\Delta j' = 0, \pm 1$ for jj-coupling. In each case, "0-0" transitions in J are not allowed.

Remarkably, selection rules for optical and for collisional transitions are not the same. For example, a mercury atom can receive enough energy in a collision with an electron to excite one of its 3P triplet states, but in the subsequent optical de-excitation, only the process $^3P_1 \rightarrow ^1S_0$ is allowed ($\Delta J = 1$). 3P_0 and 3P_2 are metastable states which are depopulated only by collisions at the sides of the tube or with other mercury atoms. (The time a mercury atom needs to travel the mean free path is small compared to the life time of those states.)

Vaporization of Liquid Mercury

The pressure of the mercury vapour plays an important role, so we need to know its value for different temperatures. The law of CLAUSIUS and CLAPEYRON³ offers the possibility to compute the pressure within the gas by means of the heat of vaporization, Q :

$$Q = T \frac{dp}{dT} \cdot (V_g - V_l). \quad (5)$$

Here, T denotes the temperature, p the pressure, $V_{g,l}$ are the specific volumina of the gaseous, resp. liquid phase. For our purpose, this law can be rewritten (inserting the particular properties of mercury) as

$$\log\left(\frac{p}{\text{Torr}}\right) = 10.55 - \frac{3333}{T/K} - 0.85 \log\left(\frac{T}{K}\right). \quad (6)$$

1.4.2 Properties of the Franck-Hertz Tube

Franck and Hertz used a vacuum tube with three electrodes ("triode"). Because of the very low pressure, some fraction of a drop of liquid mercury enclosed in the tube will immediately evaporate, so that the tube is filled with a thin gas phase of mercury. As explained in section 1.4, the anode A must be kept at a slightly negative potential compared to the grid G, such that the electrons are slowed down after passing G. Otherwise, "all" the electrons would reach A and thus contribute to the current, regardless of their energy loss within the mercury gas – no significant modulation of the current as a function of the accelerating voltage would then be observed.

The Richardson Effect

How can we produce "free" electrons which can be used to excite the Hg atoms in the tube?

Within metals, the most loosely bound electrons (valence electrons) can be displaced freely. Yet when they come close to the surface of the metal, they experience an attractive force caused by the positive charge of the ion lattice. To free an electron from the metal, one has to apply a certain amount of energy, W , ranging between 1 and 5 eV. At room temperature, the mean thermal energy of an electron, $\frac{3}{2}kT$, is much too low for it to be able to escape from the surface. This also holds if the cathode is heated. Still, because of the FERMI distribution of the electrons, some of them reach higher energies. The thermal energy is directly proportional to the temperature, so with increasing T it becomes more likely that some electrons can leave the solid material. Above the "electron sea" within the metal, a vapor of free electrons is formed; their density n in thermodynamic equilibrium can be described by the BOLTZMANN factor $\exp(-\frac{W}{kT})$. In practical use, one usually doesn't work in thermodynamic equilibrium. Instead, a strong electric field is applied to drive the electrons away from the cathode. (Otherwise, the spatial charge would keep the electrons close to the surface and eventually prevent any more electrons from escaping from the metal.)

RICHARDSON's law⁴ describes the current density of electrons leaving the metal and flowing out into the evacuated tube as a function of temperature:

$$j = e \cdot n = e \cdot C \cdot T^2 \cdot e^{-W/kT}, \quad (7)$$

where C is a constant depending on the characteristics of the metal and the form of the surface. A "good" cathode material should have a low escape energy W and a high melting point.

³cf. for example [4]

⁴cf. for example [4]

2 Setup and Realization of the Experiment

The setup of the FRANCK-HERTZ experiment was given like it is shown in figure 5. The central element of the setup is the FRANCK-HERTZ tube which consists of an anode, a cathode and a grid with different voltages and which is situated inside an oven. The oven can be heated by increasing the heating voltage. The temperature can be determined with a digital thermometer.

As cathode voltage we apply 6.3 V of alternating current from the power source. The grid voltage U_A can be varied from 0 to 50 V and is connected to a capacity and to the X-Input of the analog plotter. Like that the capacity controls the movement of the plotter in X-direction by increasing the voltage from 0 V to the adjusted maximum voltage. Simultaneously the voltage of the grid is also increased. To begin the loading procedure of the capacity, switch S_1 is opened. To reset the grid voltage the switch is closed again, shortcutting the capacity. The number of incoming electrons is determined by measuring the anode current with the help of an amplifier whose output controls the Y-deviation of the analog plotter. To modify the anode current, the back voltage U_S can be varied.

In the second part of the measurement a photomultiplier can be applied to determine the wavelength of the emitted ultraviolet light. Therefore the tube and the oven both have an UV-transparent window. The output of the photomultiplier is also connected to an amplifier. To measure the photocurrent curve, the output voltage of the amplifier is connected to the Y-channel of the plotter. Additionally a differentiation circuit can be plugged between the amplifier and the plotter.

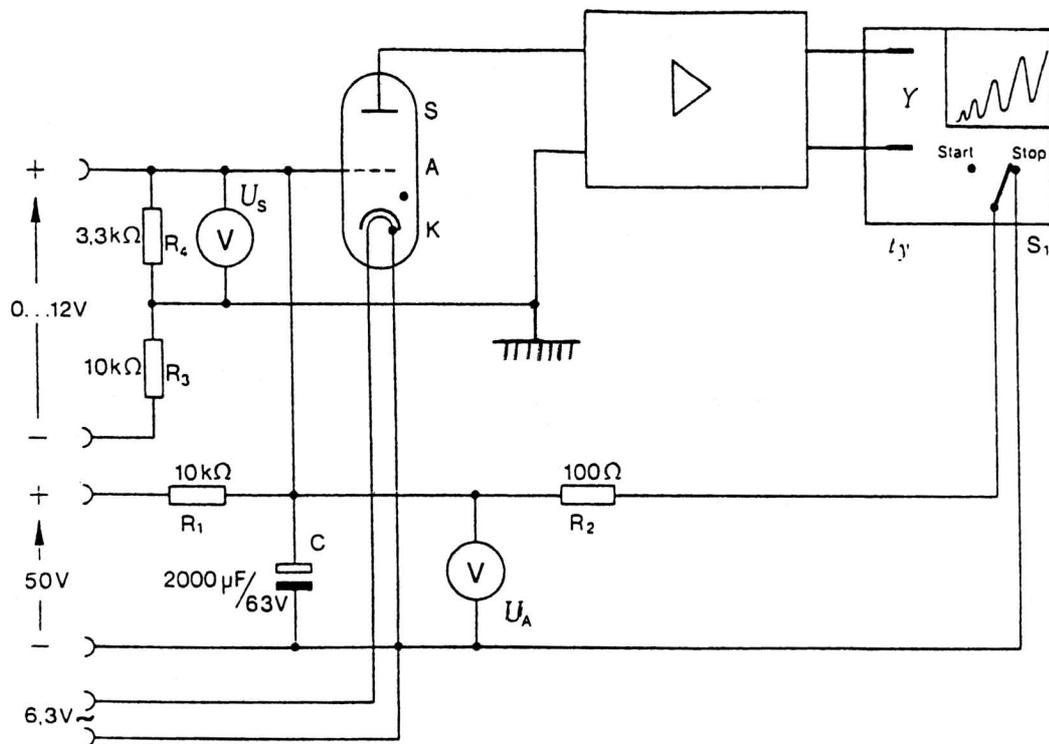


Figure 5: Sketch of the experimental setup

3 Execution of the Experiment

During the experiment we carried out two sets of measurements. First we measured the anode current and the photomultiplier current varying the back voltage, leaving the temperature fixed at 166°C . We measured 4 times both curves for the back voltages $U_{S_{max}} = (0.5 \text{ V}, 1.0 \text{ V}, 1.5 \text{ V}, 2.0 \text{ V})$. We measured the photocurrent only with the help of the differentiation circuit because of the absence of enough time. Later on it is easier to compare the results of the electronic part of the experiment with the differentiated curve than with the normal photocurrent curve.

In the second part of our experiment we measured the same curves varying the temperature of the oven. For one temperature we already had the results from the first part. Thus we only had to record the curves for three more temperatures: $T_C = (179^{\circ}\text{C}, 190^{\circ}\text{C}, 198^{\circ}\text{C})$.

In all measurements the maximum grid voltage remained constant at $U_A = 41 \text{ V}$. For each curve measurement we changed the adjustment of the amplifier and of the plotter to obtain best results. Also we varied the high voltage supply of the photomultiplier but we observed no changes or better results in our curves. To control if the measured photons are really ultraviolet light, we used a normal glass plate. With the plate in the optical path, the measured curve broke down.

During the whole experiment we had to exactly watch inside the tube to control that no blue sparking is generated between the cathode and the grid. If such a sparking starts, it is a sign for the ionisation of the mercury gas in the tube. If the gas ionises, there can happen a discharge between the grid and the cathode which can destroy the tube. To prevent the ionisation the grid voltage must immediately shut down when the blue light gets intense. At the beginning of the experiment we observed no useful results because we were too careful in increasing the grid voltage and a slight blue light could be observed.

4 Analysis

The recorded curves can be regarded in the appendix of this report.

4.1 Excitation Energy

The first task is to determine the excitation energy out of the recorded data. To get the excitation energy we have to measure the energy between the minima of the grid voltage in the curves. At the positions of the minima, the grid voltage gives the electrons enough energy to excite one or more mercury atoms. Thus the difference between the maxima exactly corresponds to the energy that leads the electron to excite one more mercury atom.

The difference between the minima can easily be converted into the excitation energy assuming that the distance between the beginning and the end of the curve correspond to the maximum grid voltage of $U_A = (41/pm0, 2)$ V. Further on assuming that the grid voltage increases linearly we can convert the distance into a voltage. Since the accelerated particles are electrons, 1 V corresponds to 1 eV.

In the following tables the results of our measurements are presented. We only measured the distances beginning from the second minimum because the position of the first minimum could not be determined exactly. In the measurement with the highest temperature there could not be determined more than one distance.

#	sheet	color	y-scale	$T/^\circ\text{C}$	U_{back}/V	minima	d / cm	U / V
1	1	blue	0.2 V/cm	166 ± 1	1.0 ± 0.1	2-3	3.8 ± 0.1	4.69 ± 0.13
2	1	blue	0.2 V/cm	166 ± 1	1.0 ± 0.1	3-4	3.9 ± 0.1	4.82 ± 0.13
3	1	blue	0.2 V/cm	166 ± 1	1.0 ± 0.1	4-5	3.9 ± 0.1	4.83 ± 0.13
4	1	blue	0.2 V/cm	166 ± 1	1.0 ± 0.1	5-6	4.0 ± 0.1	4.94 ± 0.13
5	1	black	0.2 V/cm	166 ± 1	2.0 ± 0.1	2-3	3.9 ± 0.1	4.82 ± 0.13
6	1	black	0.2 V/cm	166 ± 1	2.0 ± 0.1	3-4	3.9 ± 0.1	4.82 ± 0.13
7	1	black	0.2 V/cm	166 ± 1	2.0 ± 0.1	4-5	4.0 ± 0.1	4.94 ± 0.13
8	1	black	0.2 V/cm	166 ± 1	2.0 ± 0.1	5-6	4.0 ± 0.1	4.94 ± 0.13
9	2	red	0.2 V/cm	166 ± 1	1.5 ± 0.1	2-3	3.8 ± 0.1	4.69 ± 0.13
10	2	red	0.2 V/cm	166 ± 1	1.5 ± 0.1	3-4	4.0 ± 0.1	4.94 ± 0.13
11	2	red	0.2 V/cm	166 ± 1	1.5 ± 0.1	4-5	4.0 ± 0.1	4.94 ± 0.13
12	2	red	0.2 V/cm	166 ± 1	1.5 ± 0.1	5-6	4.0 ± 0.1	4.94 ± 0.13
13	2	green	0.2 V/cm	166 ± 1	0.5 ± 0.1	2-3	4.0 ± 0.1	4.94 ± 0.13
14	2	green	0.2 V/cm	166 ± 1	0.5 ± 0.1	3-4	3.8 ± 0.1	4.69 ± 0.13
15	2	green	0.2 V/cm	166 ± 1	0.5 ± 0.1	4-5	3.9 ± 0.1	4.82 ± 0.13
16	2	green	0.2 V/cm	166 ± 1	0.5 ± 0.1	5-6	4.0 ± 0.1	4.94 ± 0.13
17	3	red	0.1 V/cm	179 ± 1	1.0 ± 0.1	2-3	3.8 ± 0.1	4.69 ± 0.13
18	3	red	0.1 V/cm	179 ± 1	1.0 ± 0.1	3-4	3.8 ± 0.1	4.69 ± 0.13
19	3	red	0.1 V/cm	179 ± 1	1.0 ± 0.1	4-5	3.8 ± 0.1	4.69 ± 0.13
20	3	red	0.1 V/cm	179 ± 1	1.0 ± 0.1	5-6	3.8 ± 0.1	4.69 ± 0.13
21	3	green	0.05 V/cm	190 ± 1	1.0 ± 0.1	3-4	3.7 ± 0.1	4.57 ± 0.13
22	3	green	0.05 V/cm	190 ± 1	1.0 ± 0.1	4-5	3.7 ± 0.1	4.57 ± 0.13
23	3	green	0.05 V/cm	190 ± 1	1.0 ± 0.1	5-6	3.8 ± 0.1	4.69 ± 0.13
24	4	red	0.02 V/cm	198 ± 1	1.0 ± 0.1	4-5	3.8 ± 0.1	4.69 ± 0.13

Table 2: Differences between minima

With the obtained results we can calculate an average voltage. The errors are all of the same size. We have enough values to reasonable calculate a standard deviation and use it as error for the average voltage:

$$\bar{U} = (4, 80 \pm 0, 11) \text{ V.}$$

Considering the argumentation above, we easily get the **excitation energy of mercury**:

$$E_{exc} = (4, 80 \pm 0, 11) \text{ eV.}$$

In the literature we find a value for the excitation energy of mercury of 4.9 eV. This value is within our error margin. In the theoretical part we made a calculation considering all excited states of the mercury atom and got a value of 5.18 eV for the mean excitation energy (equation (4)). In this case our measured value would not be exact enough.

4.2 Wavelength of Emitted Photons

By means of the measured and calculated excitation energy of mercury we can calculate the wavelength of the emitted photons. The result should be a wavelength in the ultraviolet band. We use the formula

$$\lambda = \frac{h \cdot c}{E} \pm \frac{h \cdot c \cdot \Delta E}{E^2},$$

assuming that the h and c have no errors. We get a value of

$$\lambda = (258, 2 \pm 1, 1) \text{ nm}$$

for the wavelength of the photons. This value lays in the ultraviolet band as anticipated. It is near the theoretical wavelength for the allowed optical transition of

$$\lambda_{theo} = 253, 6 \text{ nm.}$$

The error, calculated with the GAUSS-method, seems a little small and does not include the theoretical value.

4.3 Comparison of Anode Current and Photocurrent

Due to problems at the beginning of the experiment we only acquired the photocurrent with the differentiation circuit. The differentiated photocurrent curve has its maxima and minima at the turning points of the initial photocurrent curve and vice versa. Knowing this fact we can evaluate our acquired curves without the initial photocurrent curve.

The photocurrent curve should start at the point where the first maximum of the anode current curve lays. At this point the electrons reach a sufficient energy to excite the mercury atoms. We cannot verify this fact in our curves because of the heavy disturbances in the differentiated curves. While the anode current decreases to its minimum, the photocurrent rises quickly because more mercury atoms are excited. After the minimum of the anode current, the photocurrent rises slower to reach its maximum before the anode current itself reaches its next maximum. After this maximum the photocurrent again rises quickly. There should be a point in the curves where nearly all electrons undergo elastic collisions and the photocurrent stays constant. We cannot approve such a point in our plots.

4.4 Influence of Back Voltage

In the first four measurements we analysed the influence of the back voltage on the FRANCK-HERTZ effect. To quantify the influence we examine the maximum anode currents for a grid voltage of 41 V. Additionally we measure the depth d of the fifth minimum (difference between fifth minimum and fourth maximum).

#	U_S / V	I_{max} / nA
13-16	0.5 ± 0.1	36.2 ± 0.2
1-4	1.0 ± 0.1	31.0 ± 0.2
9-12	1.5 ± 0.1	23.2 ± 0.2
5-8	2.0 ± 0.1	15.8 ± 0.2

Table 3: Influence Back Voltage

Obviously the maximum anode current decreases with increasing back voltage. We expect this behaviour because of the higher energy the electrons need to reach the anode if back voltage is increased. Therefore the current decreases.

4.5 Influence of Temperature

In the second part of the experiment we varied the temperature of the oven in which the FRANCK-HERTZ tube is situated. For the different temperatures we determine the value of the fifth maximum of the anode current because the curves are slightly shifted to the right side. The back voltage is always $U_S = 1.0 \pm 0.1 \text{ V}$. We get the results of table 4.

We observe a dramatical decrease of the anode current with rising temperature. For this observation there is no coherent explanation. The only conclusion we can get of a higher temperature is an increasing amount of inelastic collisions in the tube. Due to these collisions the electrons lose their energy not reaching enough energy to overcome the barrier to the anode. We are not able to estimate if this effect is strong enough to justify the dramatical decreasing.

#	$T / ^\circ \text{C}$	I_5 / nA
1-4	166 ± 1	23.8 ± 0.2
17-20	179 ± 1	0.99 ± 0.04
21-23	190 ± 1	0.425 ± 0.002
24+	198 ± 1	$0.00025 \pm 0,00004$

Table 4: Temperature Influence

4.6 Free Path and Life Time

The last task is to calculate the free path of 5 eV electrons for the excitation of the triplet P-level and the free path for the collision of two mercury-atoms. Additionally we shall compare this free path with the live time of the optical transition of the resonance line.

Therefore we firstly have to calculate the pressure in the FRANCK-HERTZ tube. We assume a temperature of $T = 166^\circ\text{C} = 439.15 \text{ K}$. According to equation (6) we gain a pressure of

$$p = 4.11 \text{ Torr} = 548.22 \text{ Pa.}$$

With known temperature and pressure we can calculate the free path of an electron colliding with a mercury atom:

$$\ell_{e^- \rightarrow \text{Hg}} = 71.7 \mu\text{m.}$$

The same way we calculate the free path of the Hg-atom substituting the cross-section as explained in the theory. We gain the result:

$$\ell_{\text{Hg} \rightarrow \text{Hg}} = 17.9 \mu\text{m.}$$

The mean velocity of the mercury atoms can be calculated by the BOLTZMANN distribution:

$$\bar{v} = \sqrt{\frac{8k_B T}{\pi m_{\text{Hg}}}} = 213.82 \text{ m/s.}$$

The time between two collisions of mercury atoms follows with

$$\tau = \frac{\ell}{\bar{v}} = 83.9 \text{ ns.}$$

Due to the fact that the life time of the optical transition $^3P_1 \rightarrow ^1S_0$ (117 ns) is longer than the calculated time between two collisions, most atoms will collide before losing their excitation by radiation.

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Our report contains some figures which we didn't draw ourselves. They are taken from the following sources:

- Figure 1, 2, 3 from [1];
- Figure 5 from the Advanced Laboratory Scriptum;
- Figure 4 from [3], who in turn took it from Kuhn: Atomic Spectra (1969);

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