The Franck-Hertz Experiment: Inelastic Collisions in Mercury Gas

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We have examined the behavior of low-energy electrons in a mercury gas-filled tube with various temperatures and configurations of electric fields. From the dependence of anode current upon an accelerating potential we have calculated both the ionization energy (9.84 ± 0.74 eV) and the energy of the first observed atomic transition ($6s6p\,^3P_1 \rightarrow 6s6s\,^1S_0 = 4.92 \pm 0.14$ eV) to within 1σ of the published values (10.44 eV and 4.889 eV, respectively)[1]. We also evaluated the contact potential in our tube (1.83 ± 0.35 V) which is comparable to the published value (2.1 V) in the tube documentation. In a separate study of low-energy scattering of electrons from xenon atoms, we confirm the existence of a minimum scattering cross-section at 1.1 ± 0.1 eV.

1. INTRODUCTION

In 1914, J. Franck and G. Hertz discovered that collisions between low-energy electrons and mercury atoms and discovered that these interactions occur in one of two ways: (1) the electron imparts a constant amount of energy to the atom and rebounds inelastically; or (2) the electron rebounds completely elastically, losing no energy to the atom. This discovery supported the work of Bohr and others who predicted the existence of transitions between atomic states that correspond to discrete absorption of energy.

In 1921, C. Ramsauer and J. S. Townsend independently studied scattering of electrons from noble gas atoms below the critical energy necessary to excite an atomic transition. They discovered an unexpected minimum in scattering probability at energies of ≈ 1 eV, a result that contradicted the existing classical model of this interaction which predicted atomic cross-sections independent of the energy of the incoming particle. This result required a more complete quantum model, the formulation of wave mechanics that arose several years later.

In this paper we reproduce both of these historical experiments and observe behavior that agrees with the original results. We calculate the energy of the discrete transition in mercury and confirm that it agrees with a well-known line in the mercury spectrum. We also demonstrate the existence of a scattering minimum in xenon gas at approximately 1 eV and use this result to estimate the radius of the xenon atom based on a simple one-dimensional quantum scattering model.

2. THEORY

Several types of interactions are possible between an electron and the gas atoms used in this experiment. We will discuss each of them in turn, followed by some results from kinetic theory that pertain to the experiment.

2.1. Ionization of Hg Atoms

If an electron with moderate energy (> 10 eV) strikes a neutral mercury atom, it will supply sufficient energy to completely free one of the atom's valence electrons, leaving a positively charged Hg^+ ion. Ions of this type will readily accept a free electron in a later collision, initiating a series of transitions as the electron loses energy—a process that can be readily observed as a blue glow containing many wavelengths of visible and ultraviolet light.

Since Hg^+ ions are positively charged, we can also detect the presence of ionization by applying a reverse bias that would repel electrons—the presence of a current would imply that a positively-charged particle is the charge carrier. This approach, where the onset of ionization is seen as a sudden increase in current through a gas, is the one we will use in our experiment.

2.2. Excitation of an Atomic Transition in Hg

A neutral mercury atom has 80 electrons. Shells up to $n = 4$ are completely filled with sixty electrons total, and eighteen of the remaining electrons fill $n = 5$, $l = 0, 1, 2$. The remaining two, in general, are free to transition between states—the ground state occurs with both in $n = 6$, $l = 0$.

If a low-energy electron (≈ 5 eV) strikes a neutral mercury atom, it will excite one of these electrons into a higher level. The lowest-energy transition occurs when it ends up in $n = 6$, $l = 1$ with spin opposite the electron in $6 = 5$, $l = 0$, a transition that requires 4.9 eV. (Higher energy transitions require an extra eV or two, so we will not observe those interactions.) [2] This excited state quickly decays, releasing its energy as a 2537Å photon.

Since energy transfer is limited to this one quantized transition ($6s6s\,^1S_0 \rightarrow 6s6p\,^3P_1$), we expect to see inelastic collisions only when the electron has enough kinetic energy to excite this transition. If electrons are accelerated with a constant electric field, then, we expect their energy to be a periodic function of position, with minima immediately after the electrons gain enough energy to excite mercury atoms. (see figure 1)

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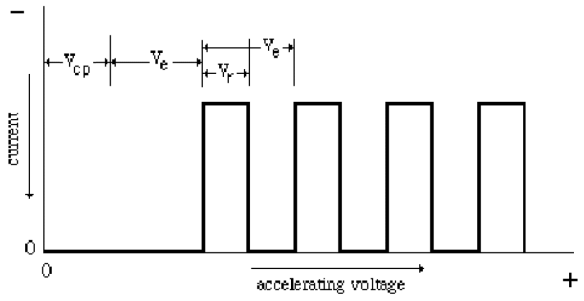

FIG. 1: Idealized model of anode current based on periodic increase in energy followed by excitation of Hg atoms

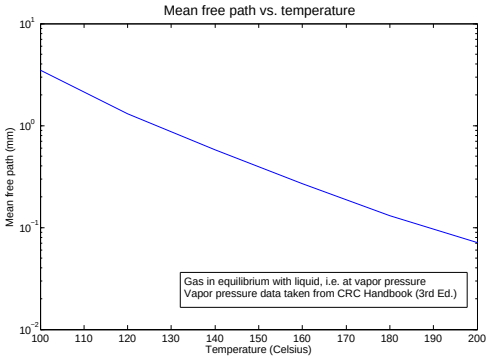

FIG. 2: Mean free path in Hg gas

2.3. Scattering of Electrons from Xe Atoms

When low energy (< 5 eV) electrons strike xenon gas, they have too little energy to excite any transition—the collision is elastic. The remarkable fact, then, is that the likelihood of scattering is strongly dependent on the momentum of the impinging electron; wave mechanics predicts that electrons with certain de Broglie wavelengths can pass directly through an atom without interacting.

A simple model with many of the same features is the transmission of an incoming plane wave from a finite square well—in this case, with width equal to the diameter of the atom.[3] If the wavelength of the electron inside the well is equal to the width of the well, then the incident wave is 100% transmitted—otherwise, there is significant scattering.

2.4. Mean Free Path in a Gas

The mean free path $\bar{\ell} = (N\sigma)^{-1}$ is defined in terms of the cross section σ of an interaction and the number density N of particles per volume. For mercury gas, this can be estimated using the vapor pressure P of mercury at temperature T and the approximate radius $a \approx 3.0$Å of the electron cloud as follows:

$$\bar{\ell} \approx \frac{k_B T}{\sqrt{2}\sigma P} \tag{1}$$

By varying the temperature of a gas, we can change the distance an electron will travel before interacting with a gas atom. This relationship is plotted in figure 2. Careful selection of this parameter will affect the energy an electron acquires between interactions, an important factor in any experiment.

3. EXPERIMENTAL SETUP

3.1. Excitation and Ionization of Mercury Vapor

The experiments with mercury gas were performed with a mercury-filled triode (manufactured by Klinger) mounted in a small oven. (see figure 3) Three voltages are applied to the tube: a filament voltage of approximately 5V to heat the cathode and allow thermionic emission of electrons, an accelerating voltage to give energy to the electrons, and a retarding voltage to repel electrons from the grid. The anode current is measured by a Keithley electrometer between the anode and cathode.

The temperature requirements are significantly different in the two experiments. For measuring excitation energy, we want to ensure that few electrons obtain enough energy to excite higher transitions; collisions should happen often compared to the distance needed to acquire 5 eV. To measure ionization, however, we *want* to obtain larger energies—so we make the path length comparable to the distance needed to acquire 10 eV. We chose to take data at 145°C (excitation) and 80°C (ionization).

We obtained ionization data by varying the voltage and taking five readings from the display on the Keithley electrometer. In order to obtain the resolution needed to see the complicated shape of the excitation curve, we connected the acceleration voltage and voltage output of the Keithley electrometer to two channels of a digital oscilloscope and recorded 2000 data points over a ten-second sweep. The analog output of the electrometer was very noisy, so we ran the signal through a simple RC lowpass filter to remove frequencies above $2\pi\,(0.1\mu F)\,(100k\Omega) = 16$ Hz.

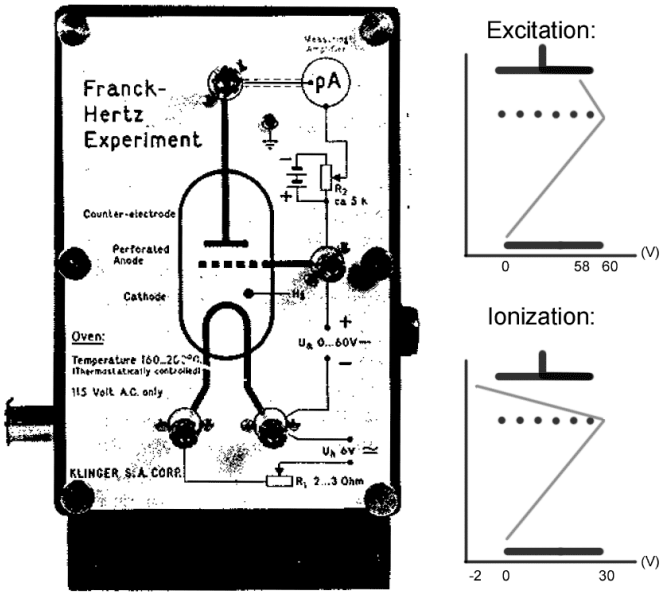

FIG. 3: Klinger tube and oven, including a schematic plot of potential in the tube. Note that the configuration for measuring ionization potentials contains a large retarding potential after the grid. Image taken from the manufacturer's specifications. [4]

3.2. Ramsauer-Townsend Scattering in Xenon Gas

The apparatus for Ramsauser-Townsend scattering consists of a 2D21 thyrotron tube with adjustable voltages similar to those seen in the Franck-Hertz experiment. The tube, however, features a "drift region" where electrons may pass through a metal box and exit via a slit at the end. If the electron is deflect by a xenon atom, it strikes the wall of the box and is not transmitted through to the anode. As in the previous experiment, we vary the density of the gas by changing the temperature—in this case by immersing the tube in liquid nitrogen. At 77K, the tube contains a negligible quantity of gas and thus the trajectory of electrons through the drift region is free of scattering.

By comparing the anode currents seen at room temperature to those seen at 77K, we can determine the effects of scattering from the gas molecules. We measured the current with a Keithley electrometer at several voltages ranging from less than a volt up to 3V, then repeated the process with the tube immersed in a liquid nitrogen bath. At each voltage we recorded five current values to obtain a measure of statistical error.

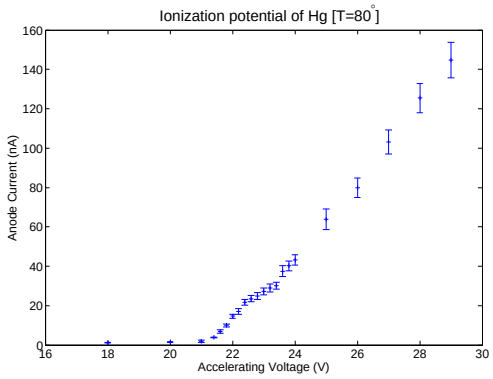

FIG. 4: Anode current in the ionization experiment.

4. DATA AND ANALYSIS

4.1. Ionization Energy

Our measurements of current in the ionization experiment are shown in figure 4. We observed two points at which the current began to increase drastically. The first corresponds to current caused by the photoelectric effect due to ultraviolet photons emitted by the gas[2]; the second, larger increase is the true ionization voltage. We estimate that this ionization begins to occur at an accelerating voltage of 23.6 ± 0.1V. This corresponds to a true potential of 21.8 ± 0.4V across the tube, correcting for the contact potential calculated from the excitation experiment. (see section 4.2) The ionization energy E_i, then, is approximately the fraction of this potential through which electrons have been accelerated. This depends on the corrected acceleration voltage V_{accel} and the length d of the path between cathode and grid as seen in equation 2:

$$E_i = \frac{V_{accel}\bar{\ell}}{d} \approx \frac{V_{accel} k_B T}{d\sqrt{2}\sigma P} \tag{2}$$

Using a temperature of $80 \pm 3°$, including an estimated uncertainty in the uniformity of temperature within the oven, we can calculate an estimate of the ionization energy. We discover that the uncertainty in the result is dominated by the uncertainty in the value of the contact potential and the uncertainty in the precise value of the tube temperature. The latter is especially significant, as the mean path length is sensitive to small fractional changes in temperature. Our final result is an ionization energy of (9.84 ± 0.74 eV), a result approximately 1σ from the published value of 10.44 eV [1].

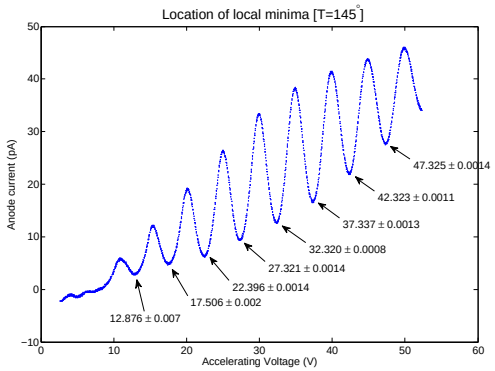

FIG. 5: Anode current in the excitation experiment. Minima are marked with values obtained from Marquadt fitting.

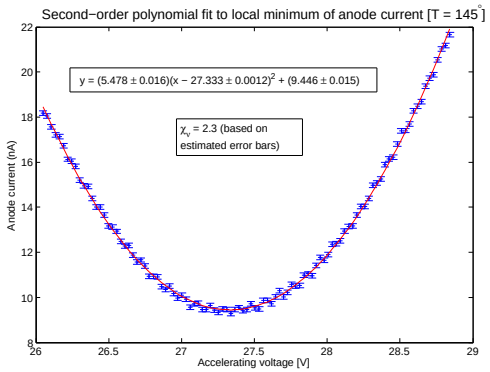

FIG. 6: Marquadt fit to a local minimum. Error bars are estimated from the electrical noise observed at constant current.

4.2. Excitation Energy

We obtained oscilloscope data that gave V_{accel} and I as a function of time. Unfortunately, the oscilloscope could not record voltages above 40V, so the acceleration voltage curve was incomplete. We fit the existing data to a polynomial to extrapolate these voltages—a quadratic term was necessary, as the ramp function was not quite linear. The resulting V-I plot is shown in figure 5.

The local minima of this curve were obtained by selecting a region $\approx$ 1V wide around each minimum and using the Marquadt routine to fit to a parabola. Error bars on the current measurements were estimated based on the electrical noise seen on the oscilloscope with constant accelerating voltage; this was approximately 1 nA. The fit to a typical peak is shown in figure 6.

By calculating a weighted mean of the difference between consecutive minima, we obtained an average value for E_0, the excitation energy necessary for the transition.

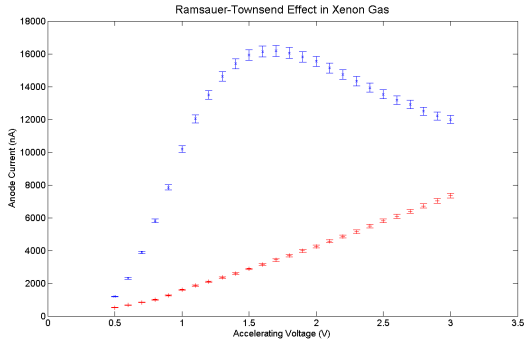

FIG. 7: Ramsauer-Townsend data collected at 77K (bottom curve) and room temperature (top curve).

Our value was 9.84 ± 0.14 eV, which is less than 1σ from the accepted value of 10.44 eV. [1] We also calculated the contact potential based on the spacings predicted from the ideal model shown in figure 1, obtaining a measurement of $V_{cp} = 1.83 \pm 0.35$V, which is consistent with the value of 2.1V given in the tube manual.[4]

4.3. Ramsauer-Townsend scattering

The data obtained from our measurements at 77K and room temperature are shown in figure 7. We observe the expected behavior—that a maximum in current (i.e. minimum in cross section) occurs at energies of approximately 1.0 ± 0.1 eV. This confirms the results demonstrated by Ramsauer and Townsend and is direct evidence against the classical model of hard-sphere scattering, which would predict no such maximum.

Acknowledgments

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[1] D. R. Lide, *CRC Handbook of Chemistry and Physics* (CRC Press, 1994).
[2] A. Melissinos, *Experiments in Modern Physics* (Academic Press, 1966).
[3] A. French and E. F. Taylor, *An Introduction to Quantum Mechanics* (Norton, 1978).
[4] K. E. P. Corp., *Franck Hertz Experiment*.