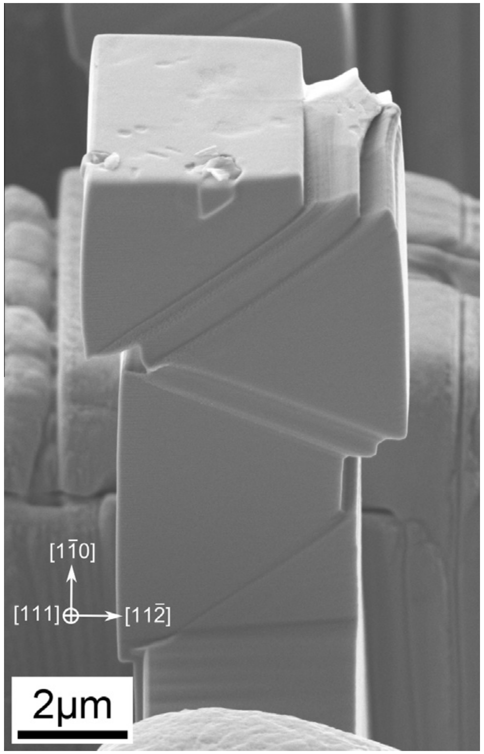
Dislocation avalanches: experiments and simulations

Péter Dusán Ispánovity

Eötvös University Budapest

Motivation

- Large stochastic fluctuations arise during the deformation of small (few um) crystalline samples
  - Makes predictable deformation impossible
- Power-law distributions characterize intermittency
- The question of universality is still open
  - What physical processes affect the critical behaviour and how?



Outline

- Intermittency in experiments on crystalline plasticity
  - Acoustic emission
  - Micropillar experiments
  - Nanoindentation pop-in events
- Simulations
  - Discrete dislocation dynamics
  - Stochastic models
- Simple 2D dislocation model
  - Anomalous system size behaviour
  - Extended cryticality

Crystal plasticity

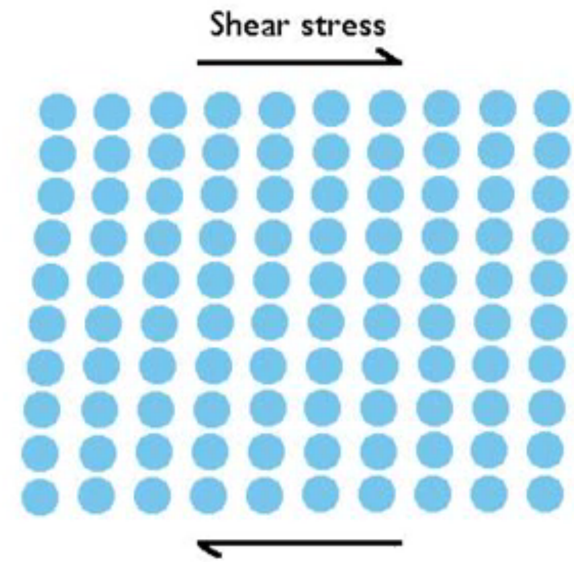
Dislocations

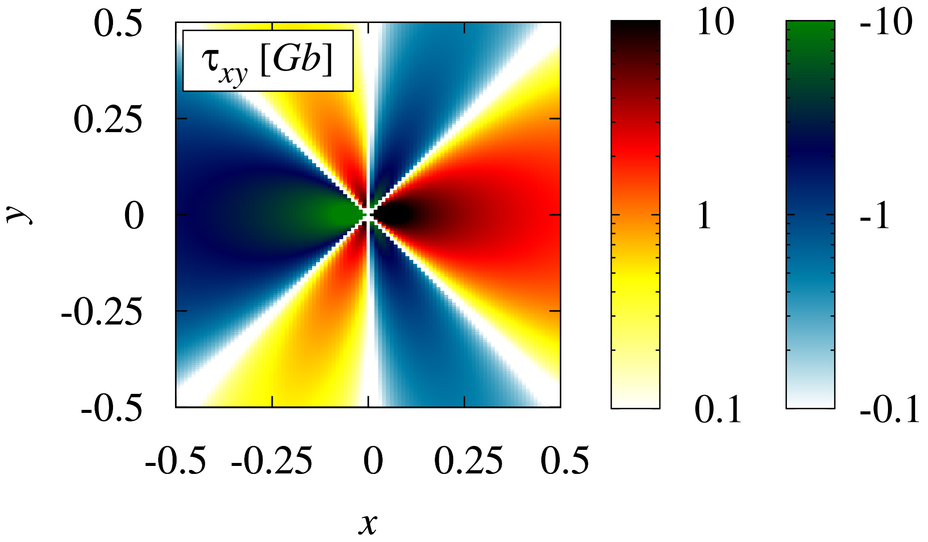
- Linear lattice defects
- Carriers of plasticity
- They move due to shear stress
- They generate long-range shear stress field:

$$\sigma_{xy}(\boldsymbol{r}) = Gb\frac{x(x^2-y^2)}{(x^2+y^2)^2}$$

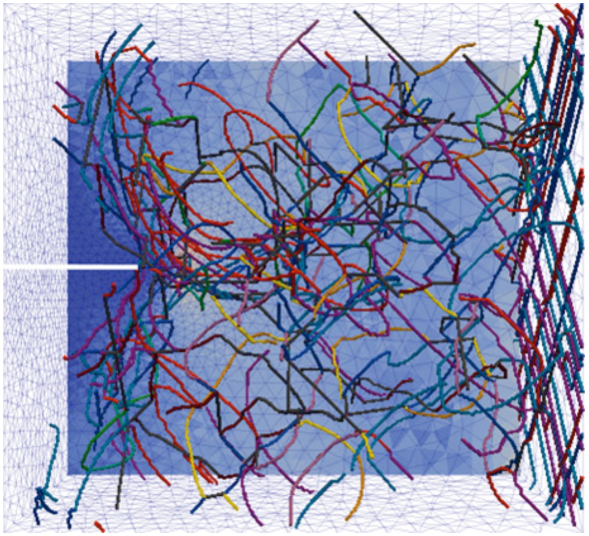
$$= Gb\frac{\cos(\varphi)\cos(2\varphi)}{r}$$

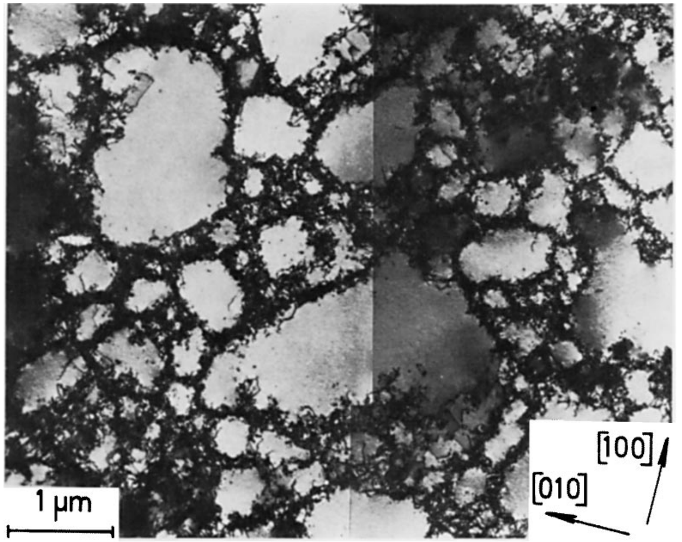
- Complex spatio-temporal dynamics





Dislocation patterns





- Multitude of observed dislocation patterns
  - Fractal, periodic, etc.
  - Crucial impact on plastic properties
  - May lead to strain localization and failure of the material

H Mughrabi, F Ackermann, K Herz, STP675, 1979

H Mughrabi, T Ungár, W Kienle, M Wilkens, Philos Mag, 1986

Relevant lenght scales

- Atomic scale
  - Dislocation reactions and mobility
- Dislocation pattern
  - Scale proportional with $\varrho^{-1/2}$: principle of similitude
- Quenched disorder
  - Precipitates, point defects (solute atoms, vacancies, etc.)
- Grain size
  - Dislocations typically cannot penetrate boundaries
- Specimen size

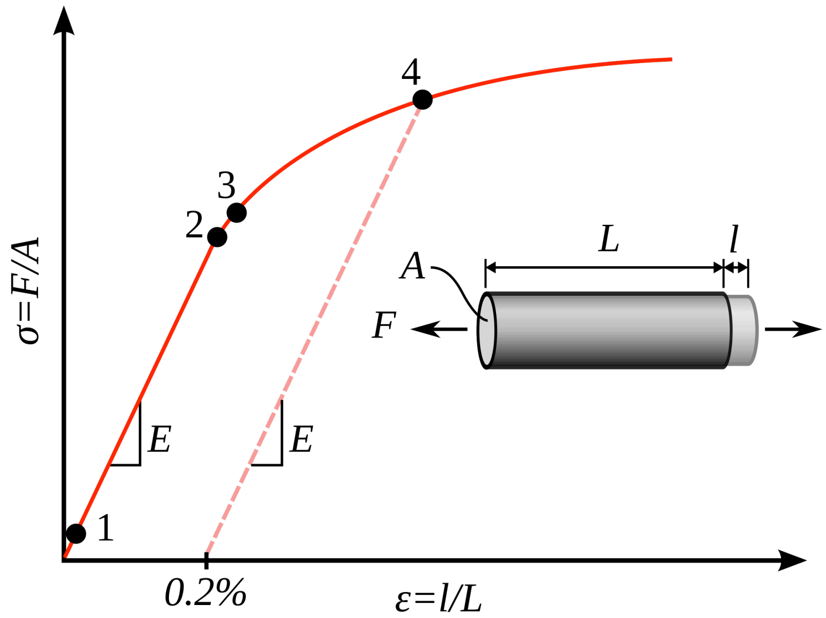
Small scales typically lead to hardening of the specimen

Comments

- Thermal activation is not relevant at low temperatures
- Dislocation motion is dissipative

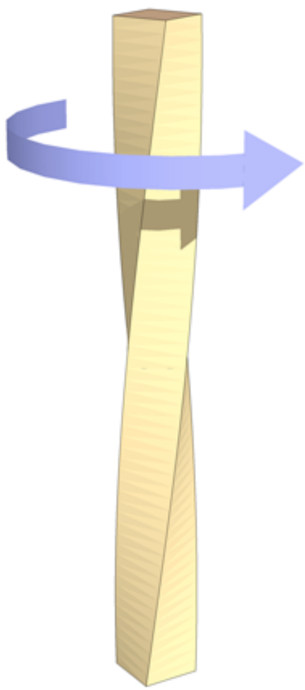
Stress-strain response

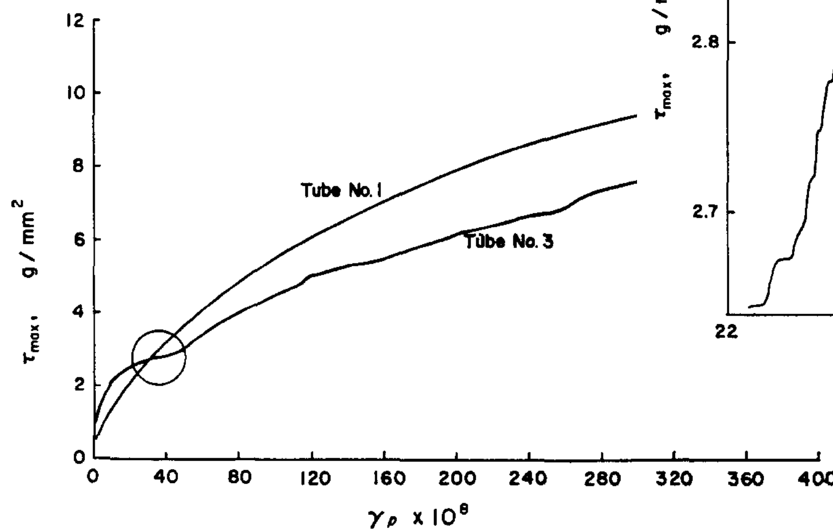
- Smooth curve for bulk samples
  - Large number of dislcoations
- Microplasticity
  - Deformation before yield
- Work hardening
  - Dislocations may multiply, annihilate and react during deformation
  - Pattern formation

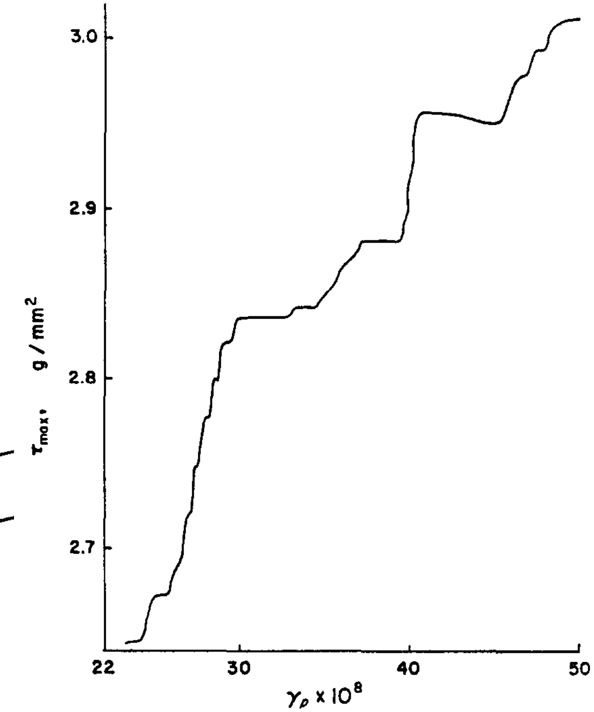


Microplasticity

- Torsion of Cu (fcc) and Zn (hcp) tubes
- Deformation sensitivity: 10^{-9}







R.F. Tinder, J. Washburn, Acta Metall. (1964)

R.F. Tinder, J.P. Trzil, Acta Metall. (1973)

Crackling during bulk plasticity

- Zn single crystal (hcp)
- ‘the strain curve of zinc crystals is not smooth, but consists of individual more or less sharp cracks’

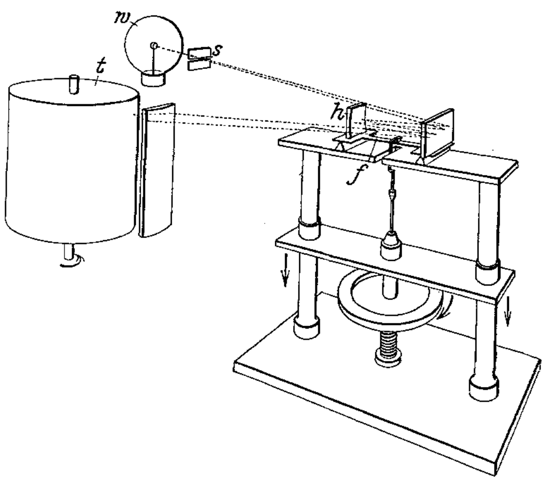


Fig. 1. Die Versuchsanordnung.

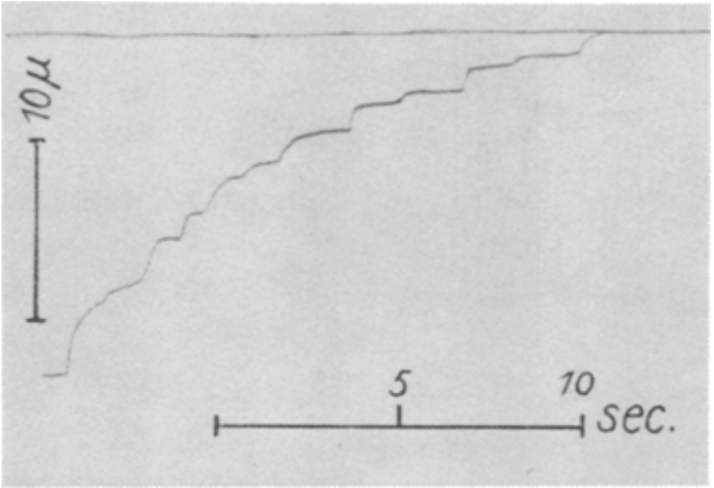
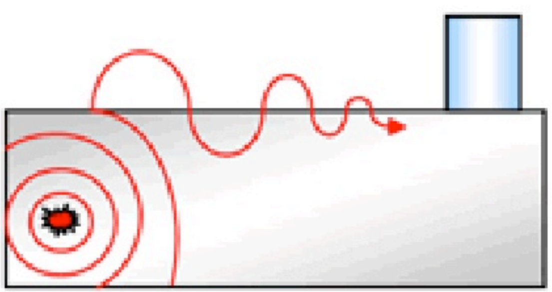


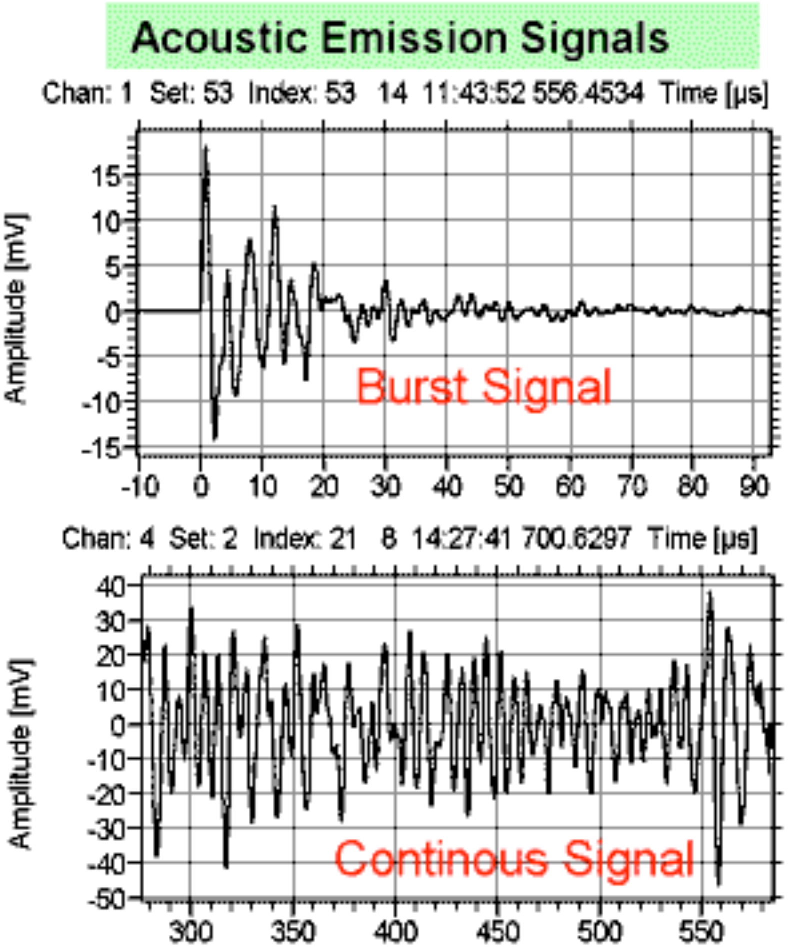
Fig. 2. Sprunghafte Dehnung eines Zinkkristalls bei 80° C.

[Becker, Orowan (1932)]

Acoustic emission

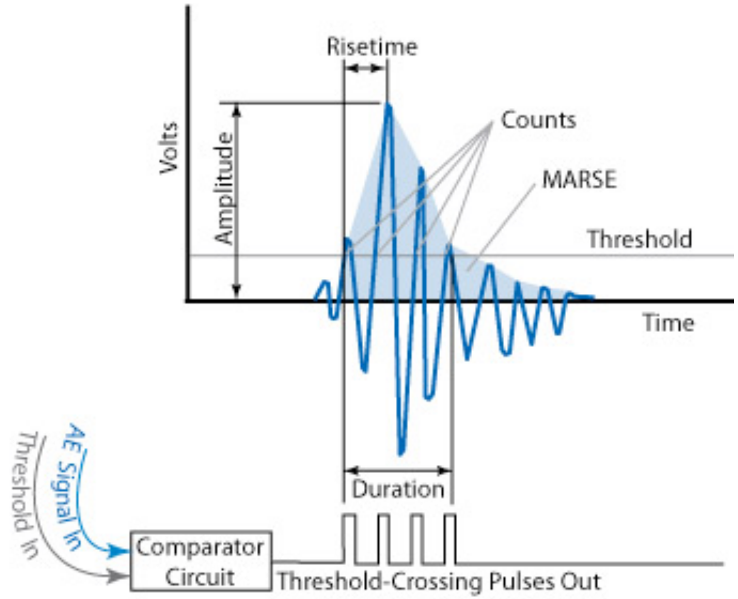
- Detection of acoustic waves on the surface using a piesoelectric detector
  - The detected signal is characteristic to the source as well as the detector
  - The distance of the source matters
  - Burst-like vs. continuous signal

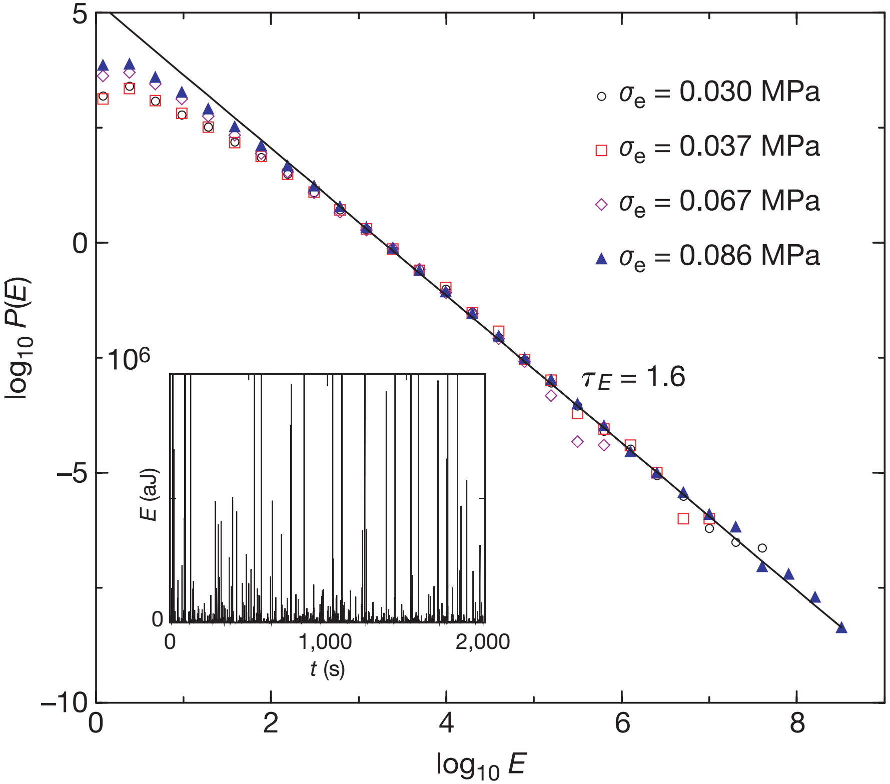




AE of ice

- Creep of ice (less than 0.1 MPa)
  - $P(E) \propto E^{-\tau_E}$
  - $\tau_E \approx 1.6$ (signal energy)
  - $\tau_A \approx 2.0$ (signal amplitude)
  - No cut-off

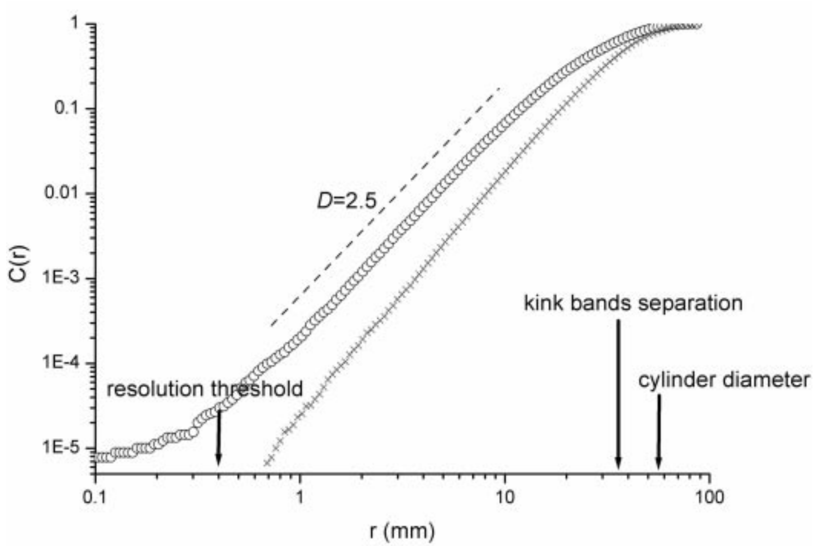


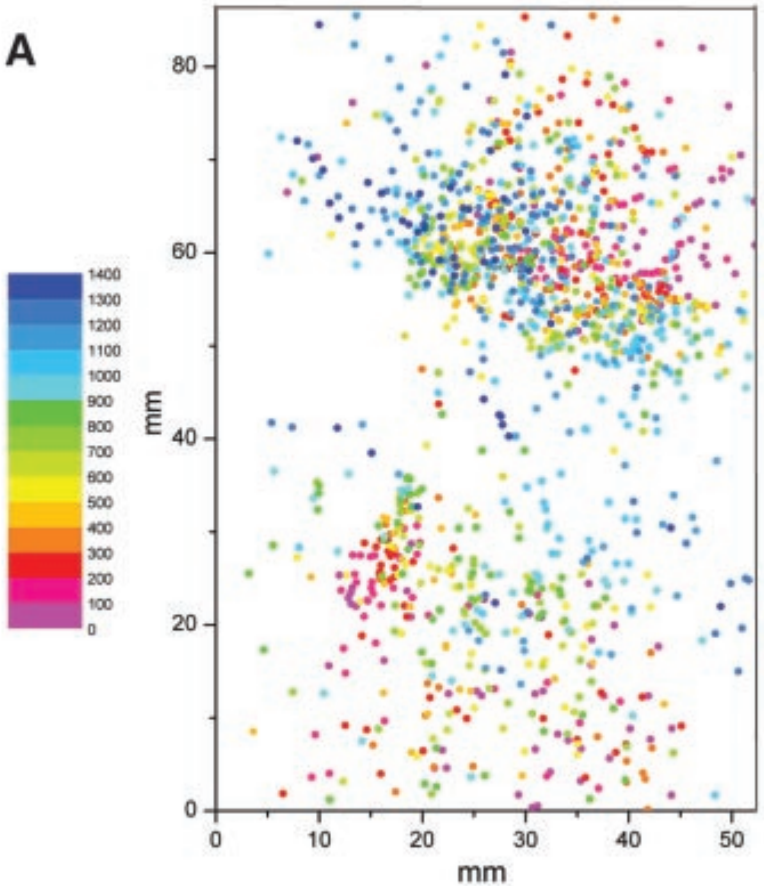


[Miguel *et al.*, Nature (2001)]

Spatial clustering of avalanches

- With more (4) detectors the origin of the acoustic signal can be located
  - They originate from a fractal sub-domain: $D = 2.5$

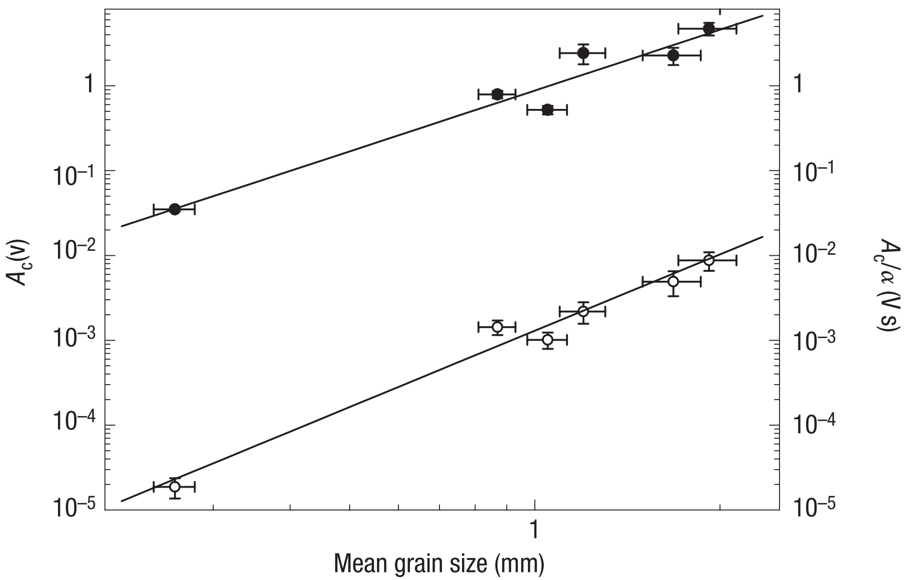


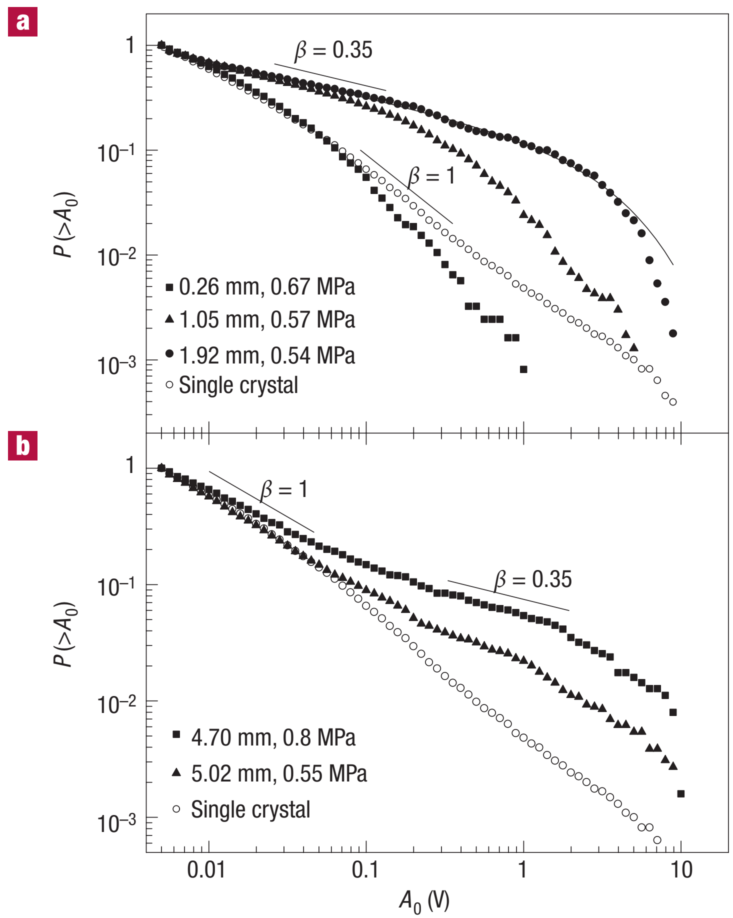


[Weiss *et al.*, Science (2003)]

Effect of grain size

- Polycrystalline ice
  - Cutoff decreasing with grain size
  - Bimodal distribution due to secondary triggering across boundaries

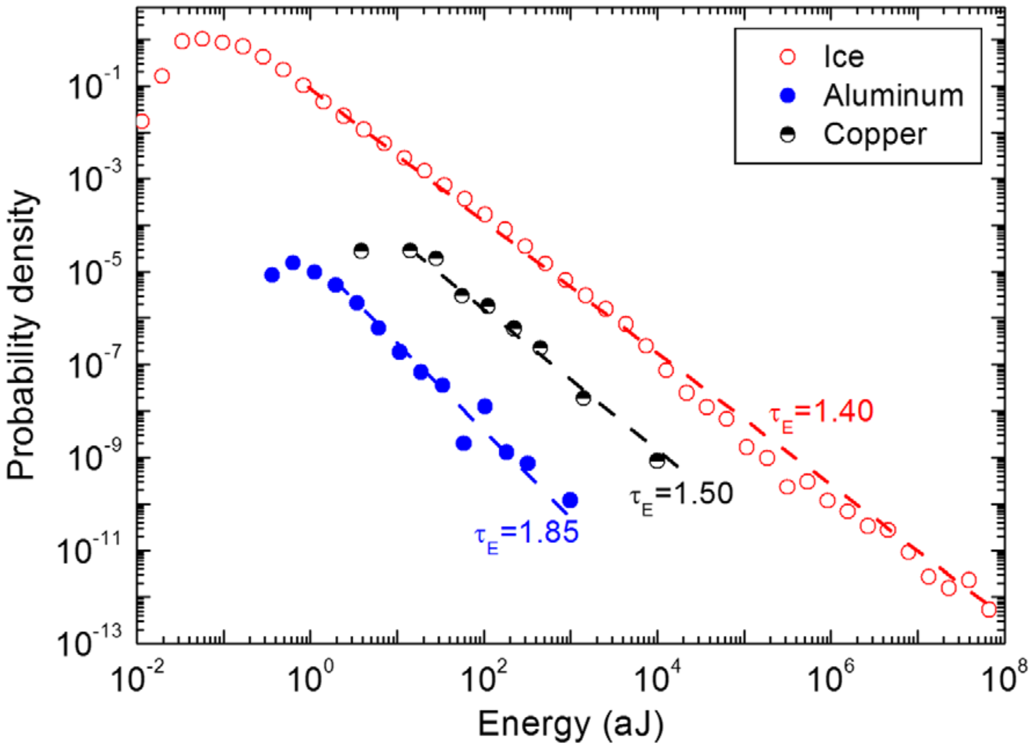




[Richeton, Weiss, Louchet, Nat. Mater. (2005)]

HCP vs FCC materials

- Acoustic signal of fcc materials is mostly continuous interrupted with only few bursts
  - In fcc materials forest dislocations and junctions introduce a length scale: mild Gaussian fluctuations
  - In hcp no such scale exists: wild power-law fluctuations



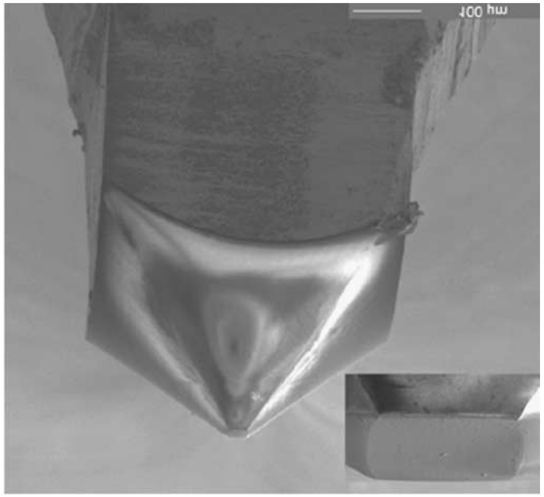
[Weiss *et al*, PRL (2015)]

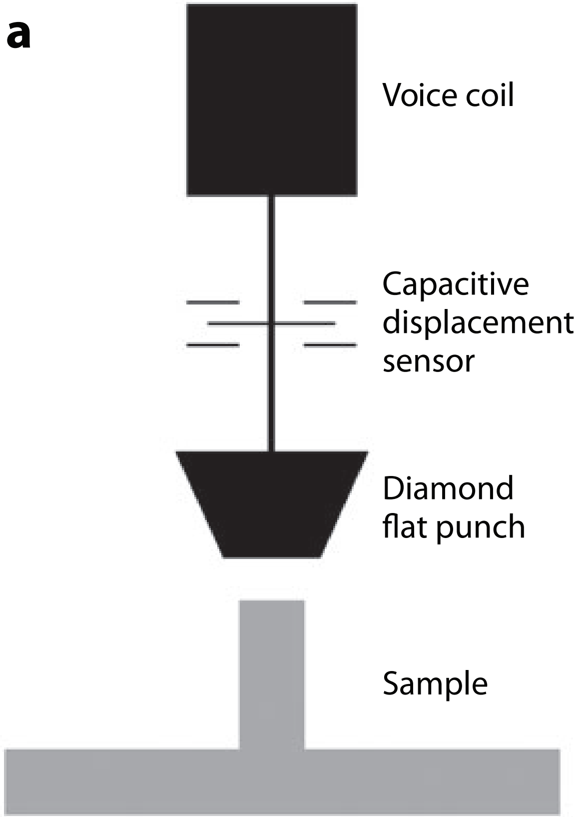
Conclusions of AE

- Relation between measured quantites and parameters of strain bursts somewhat unclear
- Time resolution for large samples or large strain rates may be a problem.
- Clear sign of criticality for bulk samples
  - Exponents suggest non-universal behaviour for different materials
- Internal length scales
  - No cutoff is detected for single crystals - no information on length scale
  - System size dependence has not been studied – AE only detectable for large samples

Micropillar compression

- Compression using a nanoindentor device
- Flat punch diamond tip
- The device contains a spring for displacement measurement
  - Machine stiffness comparable to sample stiffness



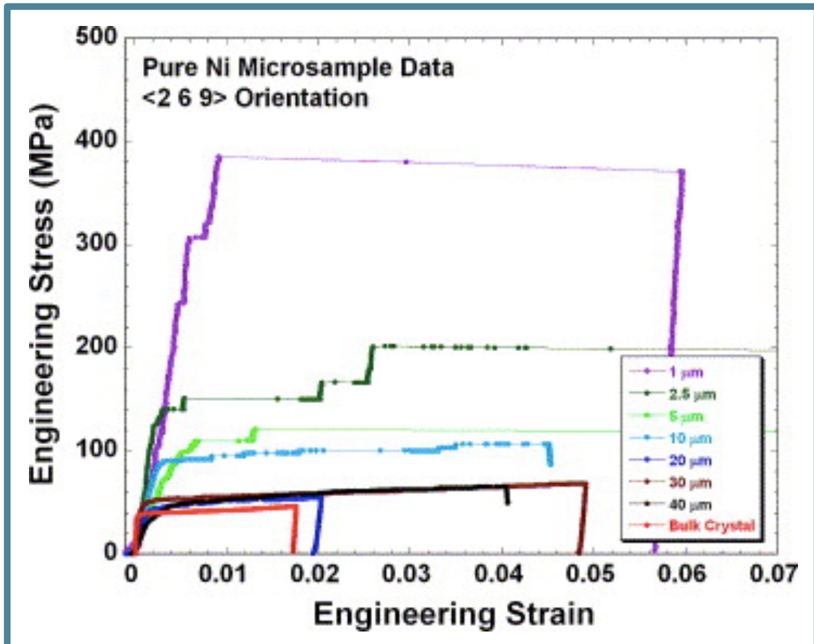


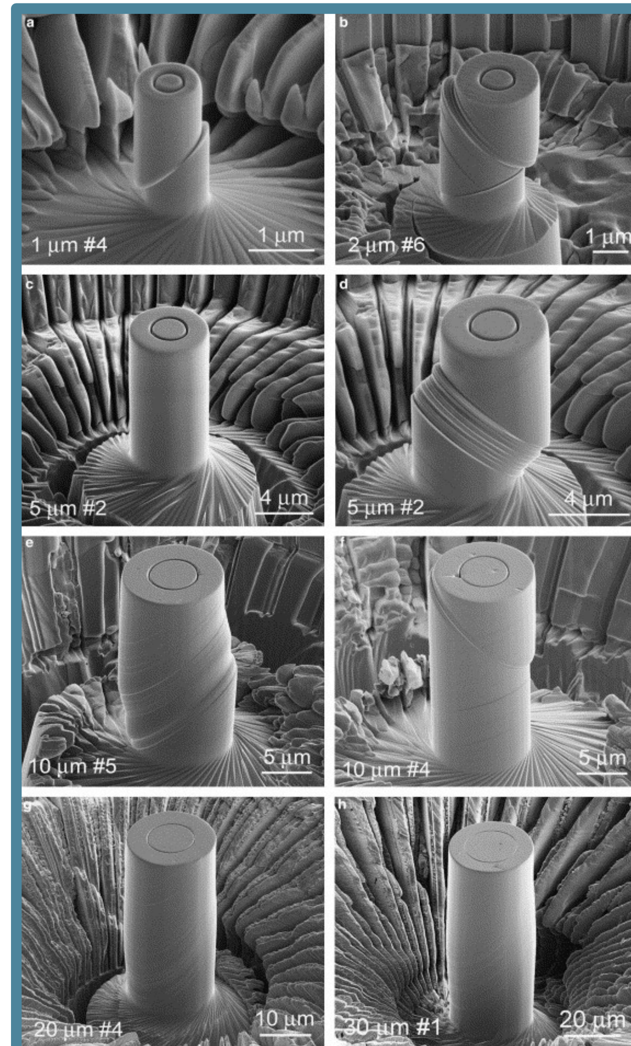
Micropillar compression

- Zn pillar oriented for single slip
- Diameter: 8 um
- Height: 24 um
- Displacment rate: 10 nm/sec
- Sudden activity on different slip planes (strain bursts, dislocation avalanches)
  - Unpredictable and localized both in time and space

Micron-scale crystal plasticity

- Size effects: smaller pillars are harder
- Stochasticity: smaller pillars are more random

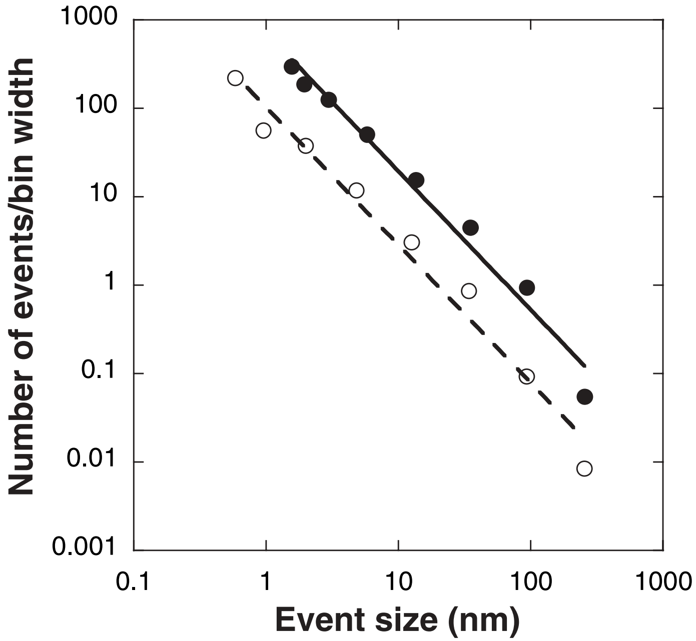




D. M. Dimiduk *et al.*, Acta Mater., 2005

Strain burst statistics

- Previous investigations
  - Strain burst sizes are power-law distributed:
    - $P(s) \propto s^{-\tau} f(s/s_0)$
    - τ: avalanche exponent
    - s_0: avalanche cut-off
    - f: cutoff function (e.g. $f(x) = \exp(-x)$)
  - Experimentally measured exponents:
    - $\tau \approx$ 1.5, 2.0, 1.3



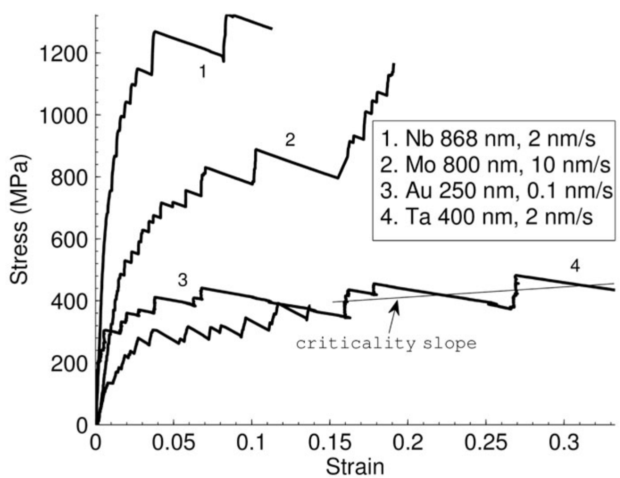
Dimuduk *et al.*, Science, 2006

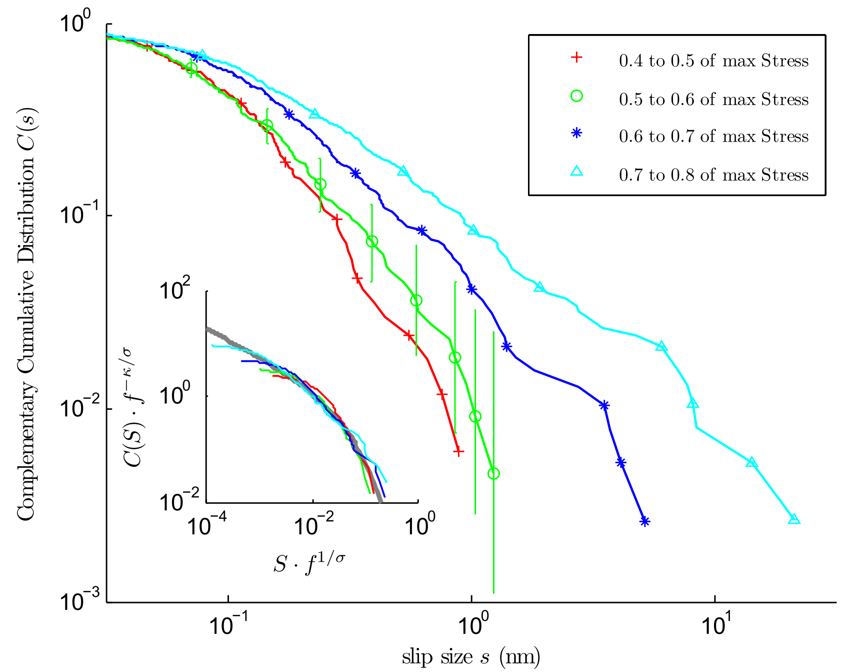
Friedman *et al.*, PRL, 2012

Zhang *et al.*, EPL, 2012

Mean-field depinning

- Compression of 7 Mo pillars
  - Diameter: 800 nm
- Stress-binned statistics
- Scaling function:
  - $D(S,\tau) \sim S^{-\kappa} f_S(S(\tau_c - \tau)^{1/\sigma})$
  - Mean field exponents obtained

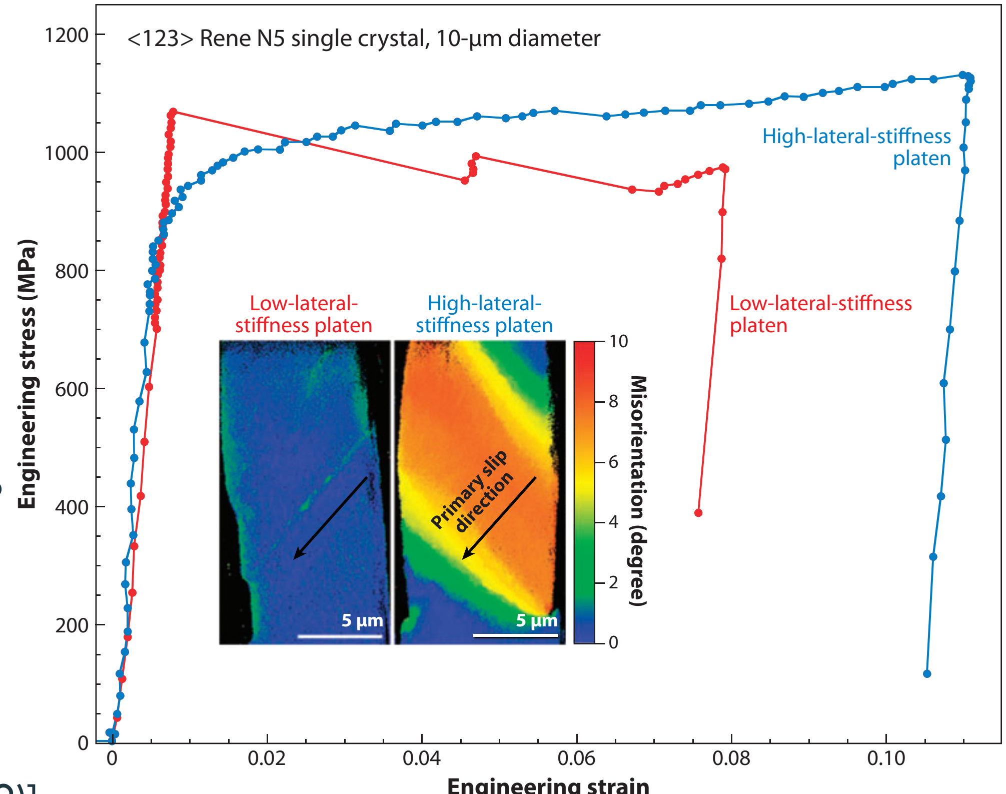




[Friedman *et al*, PRL (2012)]

Issues with micropillar experiments

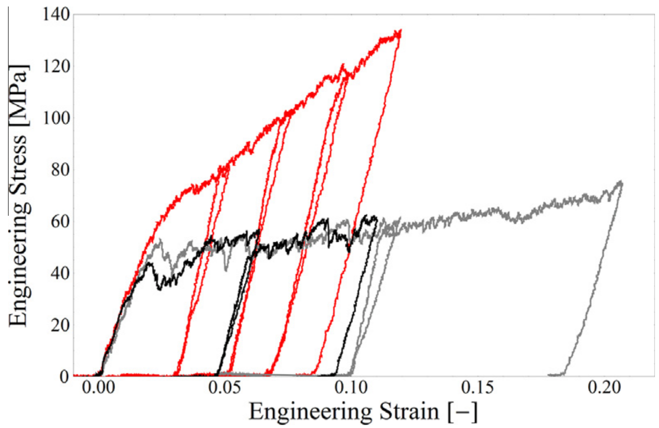
- Small number of events: large statistical errors
- Yield stress of individual pillars strongly fluctuates
- Very few avalanches in the elastic regime
- The results are very sensitive to experimental details
  - System size
    - For nanopillars new defromation mechanisms
  - Parallel tip and sample surface
  - Surface degradation duew to FIB
  - Deformation and sampling rate
  - Non-uniform pillar shape
  - Machine stiffness

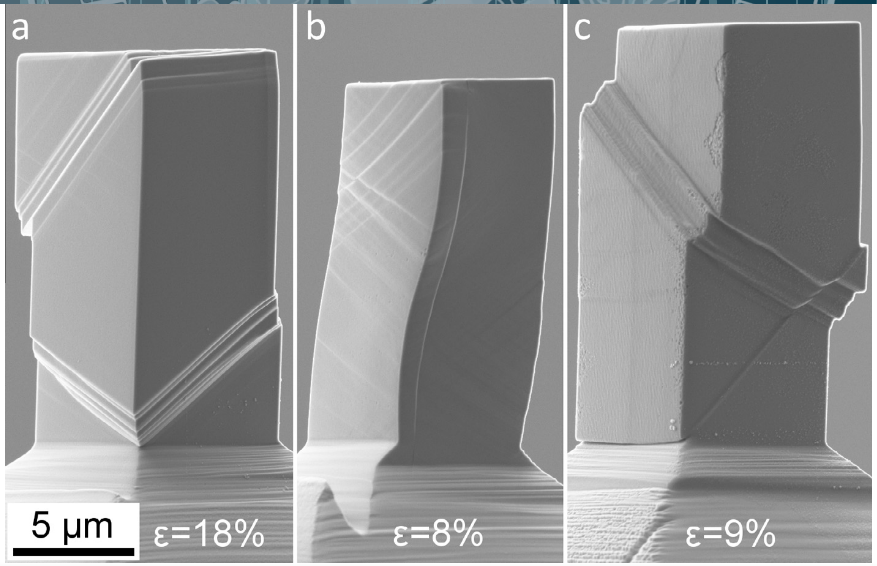


[Uchic, Shade, Dimiduk, Ann. Rev. Mater. Sci. (2009)]

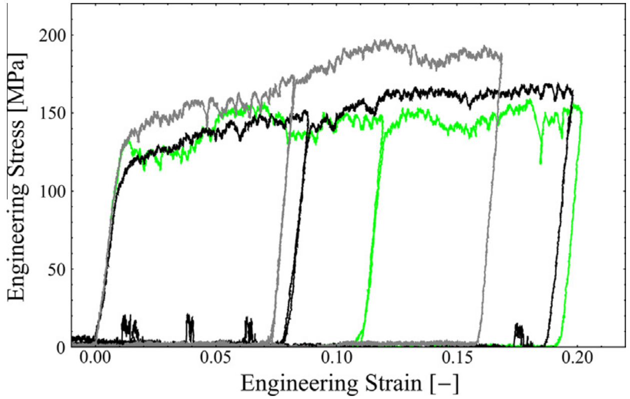
Role of grain boundaries

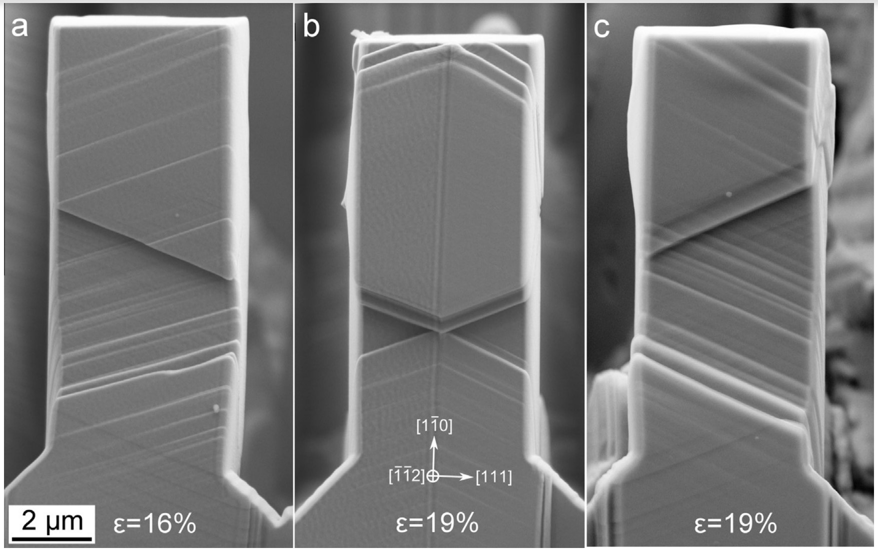
Low angle grain boundary





Coherent grain boundary

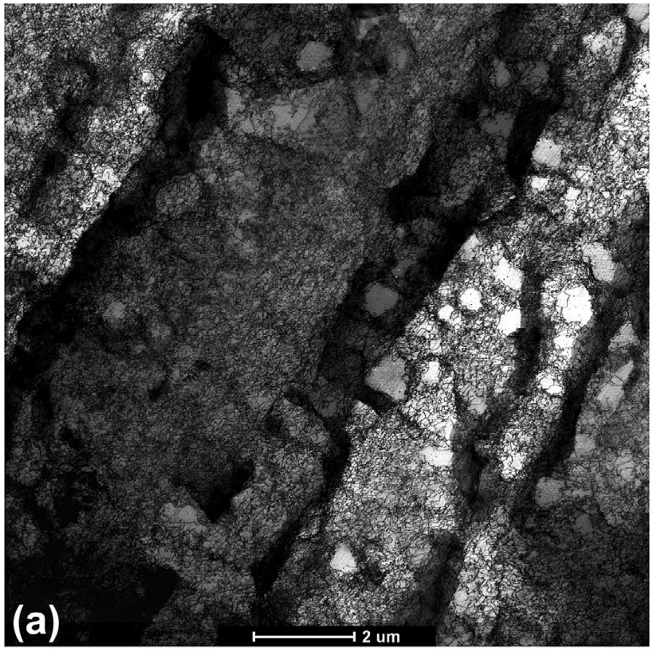


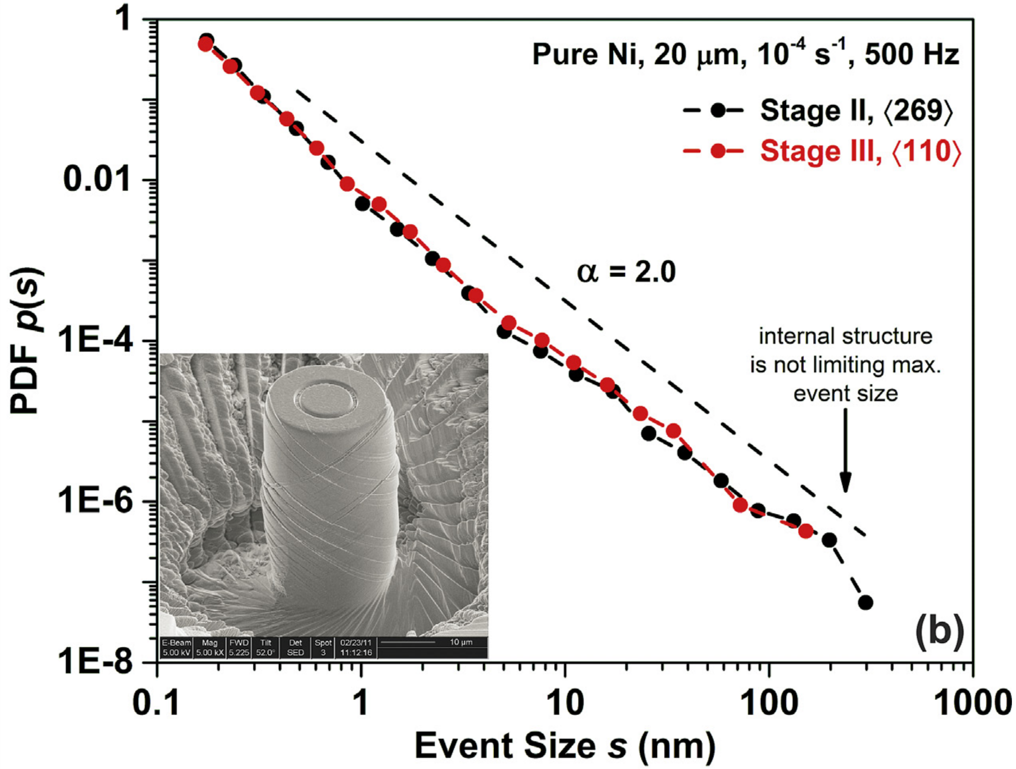


[Imrich *et al.*, Acta Mater. (2014)]

Effect of microstructure

- Avalanche distribution unaffected by original dislocation structure
- Sampling rate affects the exponent

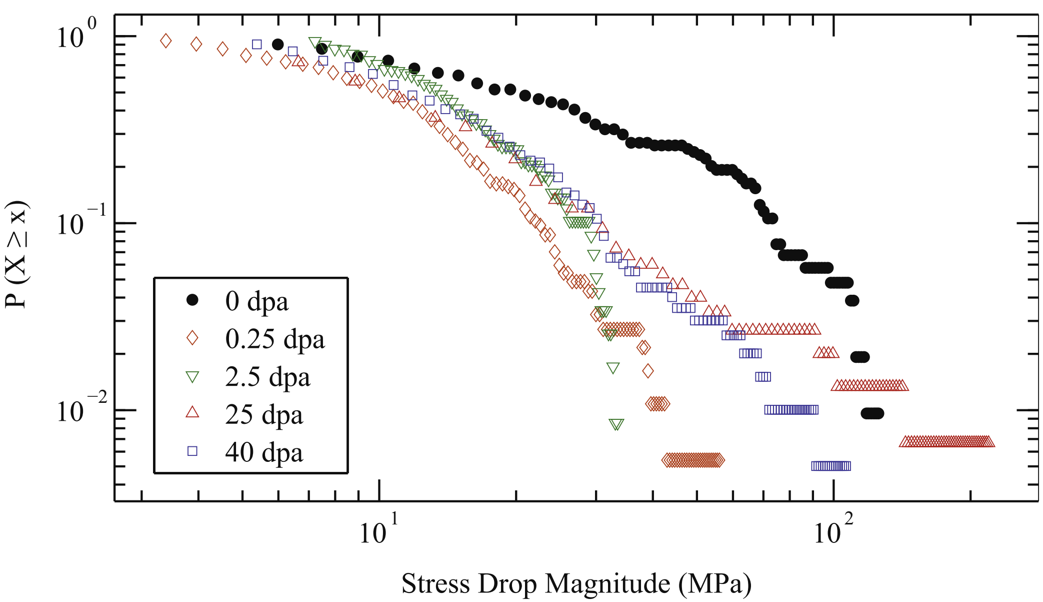


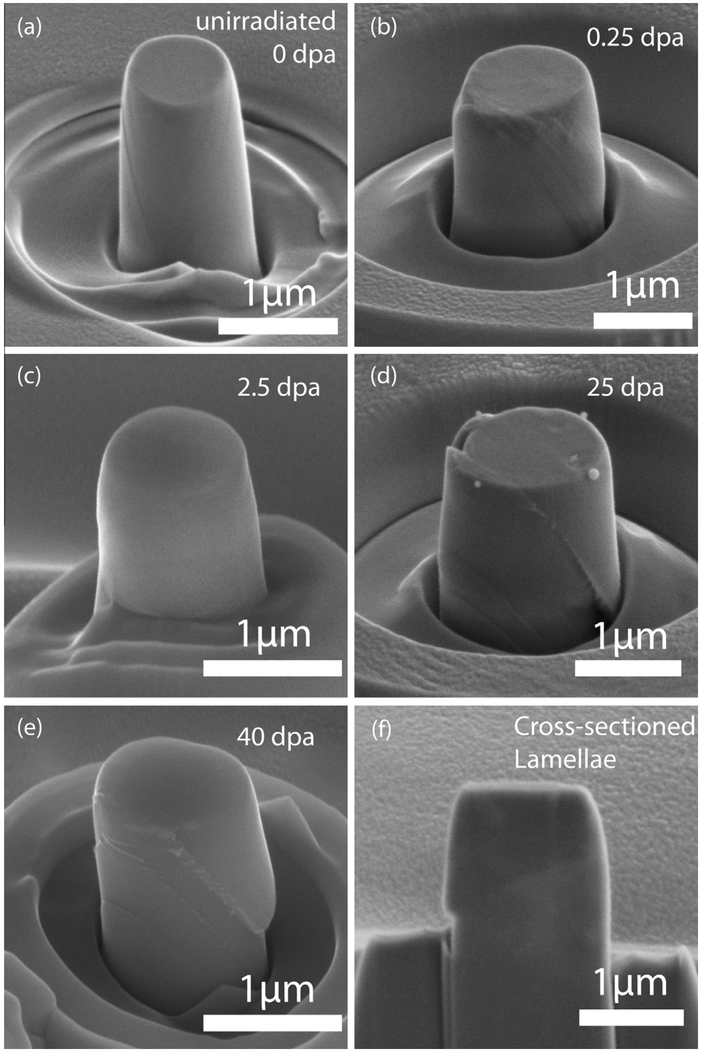


[Maass, Derlet, Acta Mater. (2018)]

Effect of irradiation

- Self-ion irradiated <111> Ni:
  - Smoother response for small doses
  - Large fluctations for high doses

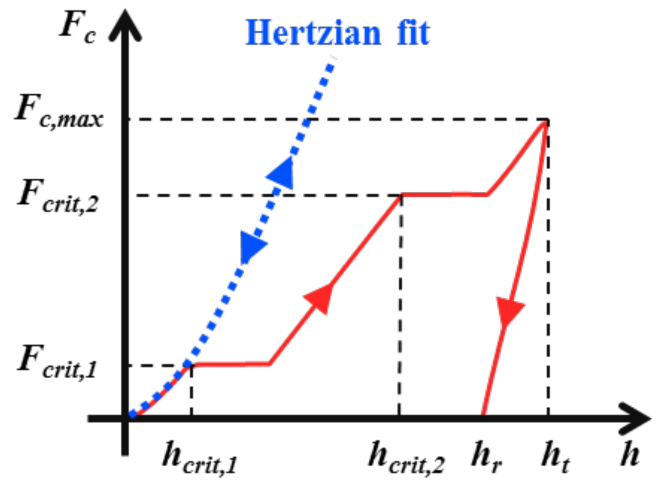


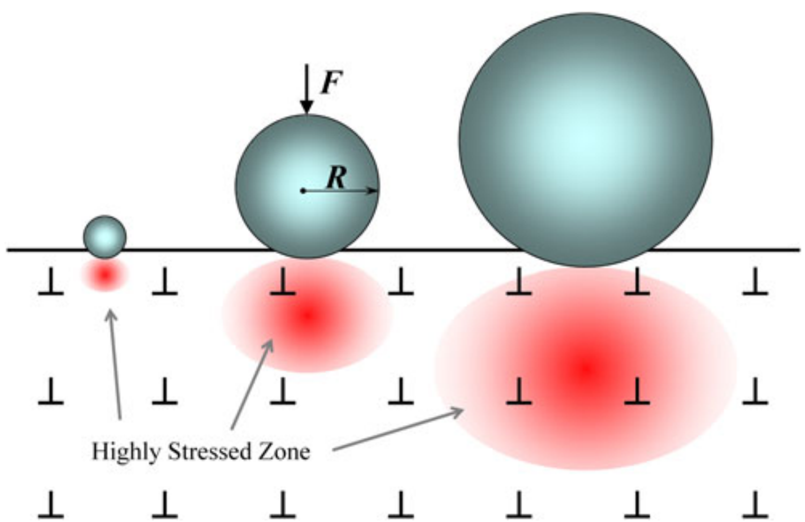


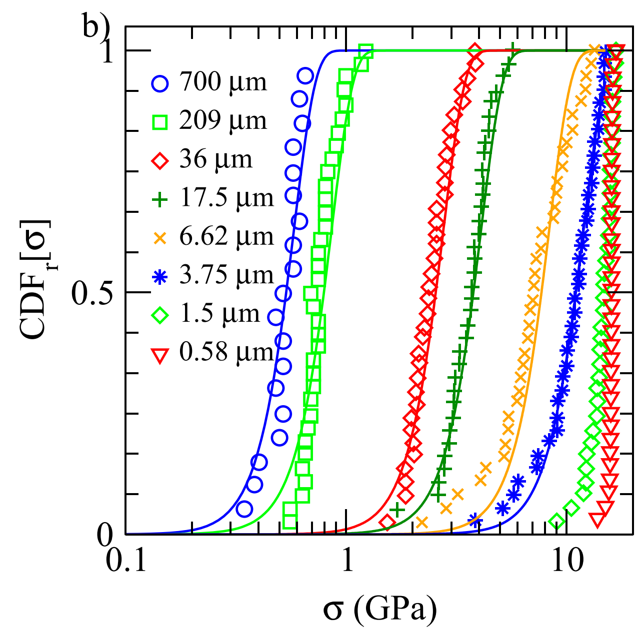
[Zhao *et al*, Acta Mater (2015)]

Nanoindentation: pop-in events

- First pop-in events in Mo
- Weibull distribution describes well the pop-in stresses







[Phani, Johanns, George, Pharr, J. Mater. Res. (2013); Derlet, Maass, J. Appl. Phys. (2016)]

Conclusions of experiments

- Clear sign of critical behaviour
  - Power-law distributions
  - Technologically important to understand small-scale deformation
- Internal scales seem to affect the avalanche distribution
  - Grain boundaries, quenched pinning
- Measured data suffer from
  - Small datasets
  - Experimental difficulties
  - Too small samples
- One cannot conclude regarding universality

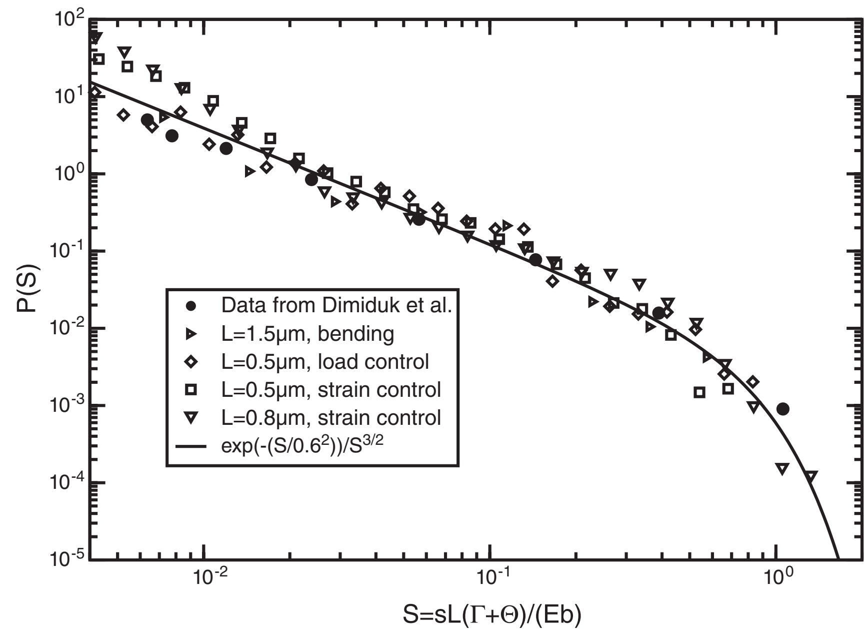
Discrete dislocation dynamics (DDD)

- 3D DDD simulations on Al
- Mean-field exponent
- Behaviour of the cut-off:

$$s_0 \propto \frac{bE}{L(\Theta + \Gamma)}$$

  - b: Burgers vector
  - E: Young modulus
  - L: system size
  - Θ: strain hardening
  - Γ: effective stiffness of the machine-sample system

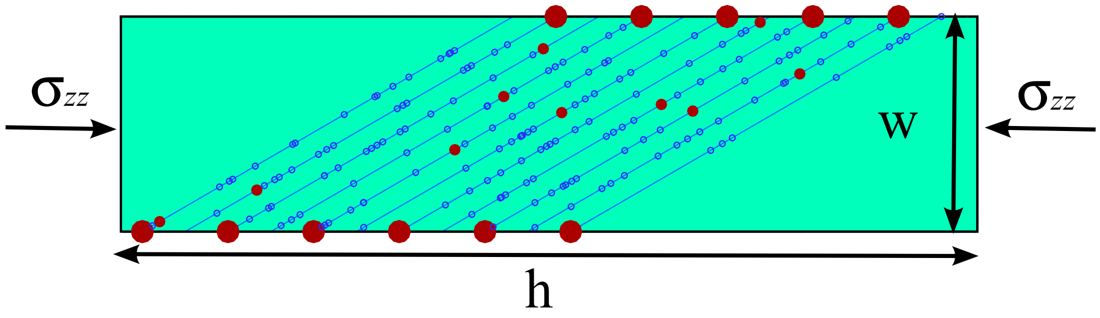
$$P(s) \propto s^{-\tau} f(s/s_0)$$



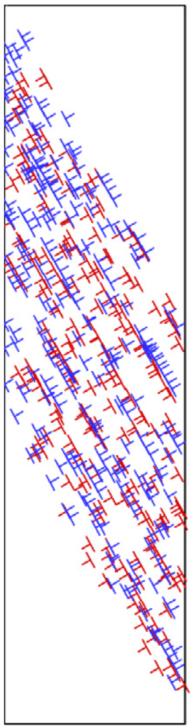
[Csikor *et al*, Science, (2007)]

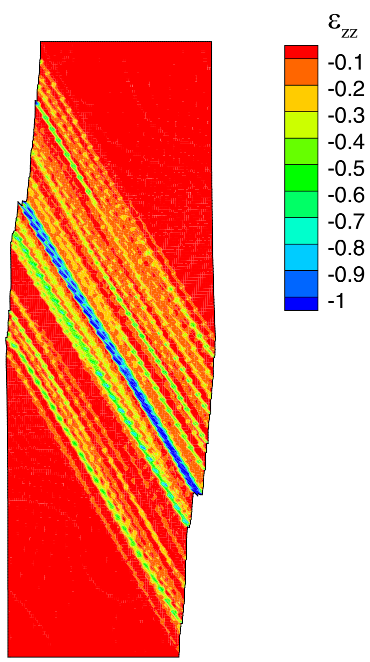
2D DDD

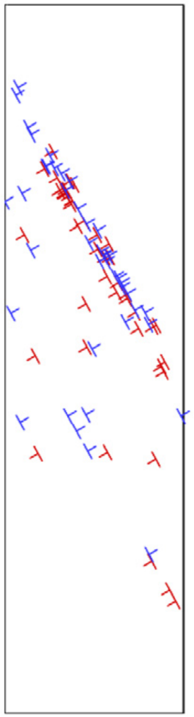
- Edge dislocations in single or multiple slip systems
  - Sources and obstacles

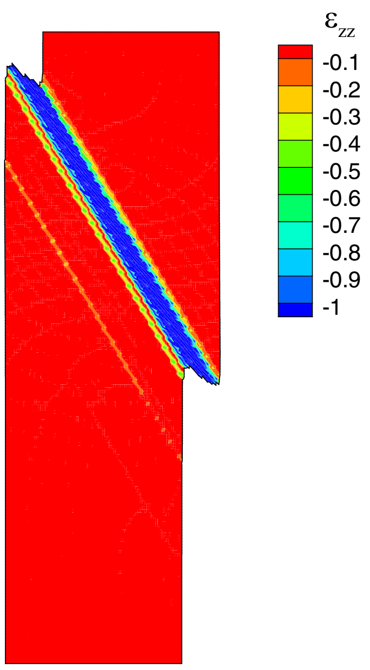


[Papanikolaou, Song, van der Giessen, J. Mech. Phys. Solids (2017)]



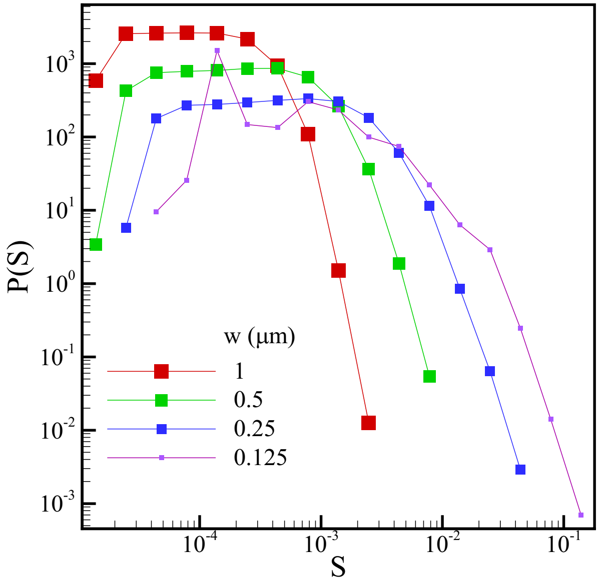


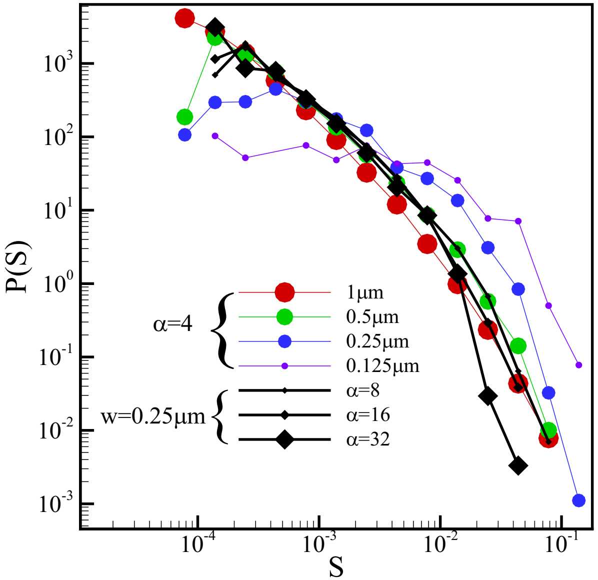




2D DDD

- Weak sources (left): cutoff dependent on sample size
  - Size effect; larger events for thin samples, $\tau = 1.5$
- Strong sources (right): constant cutoff
  - No size effect; event distribution independent of size; $\tau = 1.9$

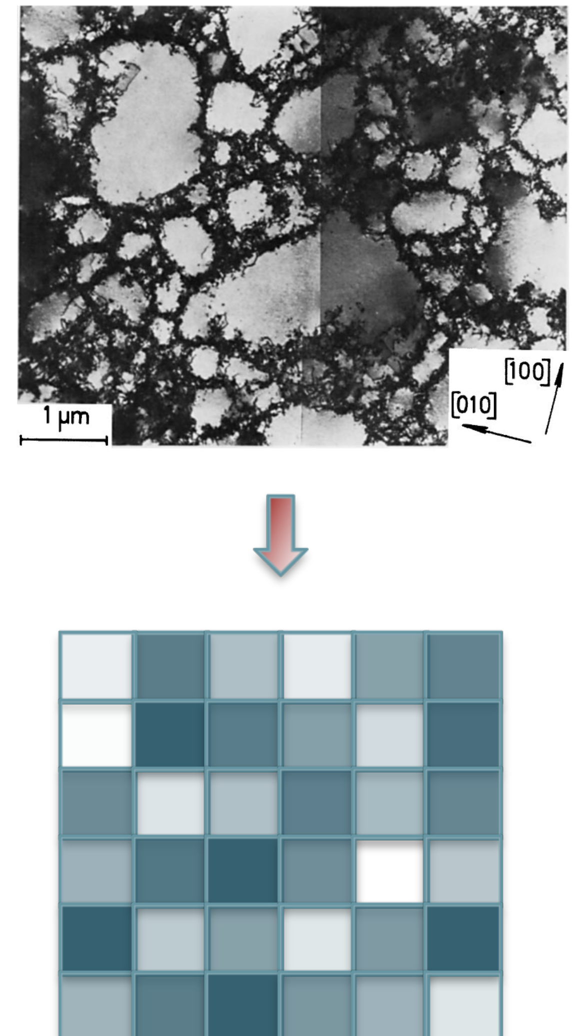




[Papanikolaou, Song, van der Giessen, J. Mech. Phys. Solids (2017)]

Representation of inhomogeneities

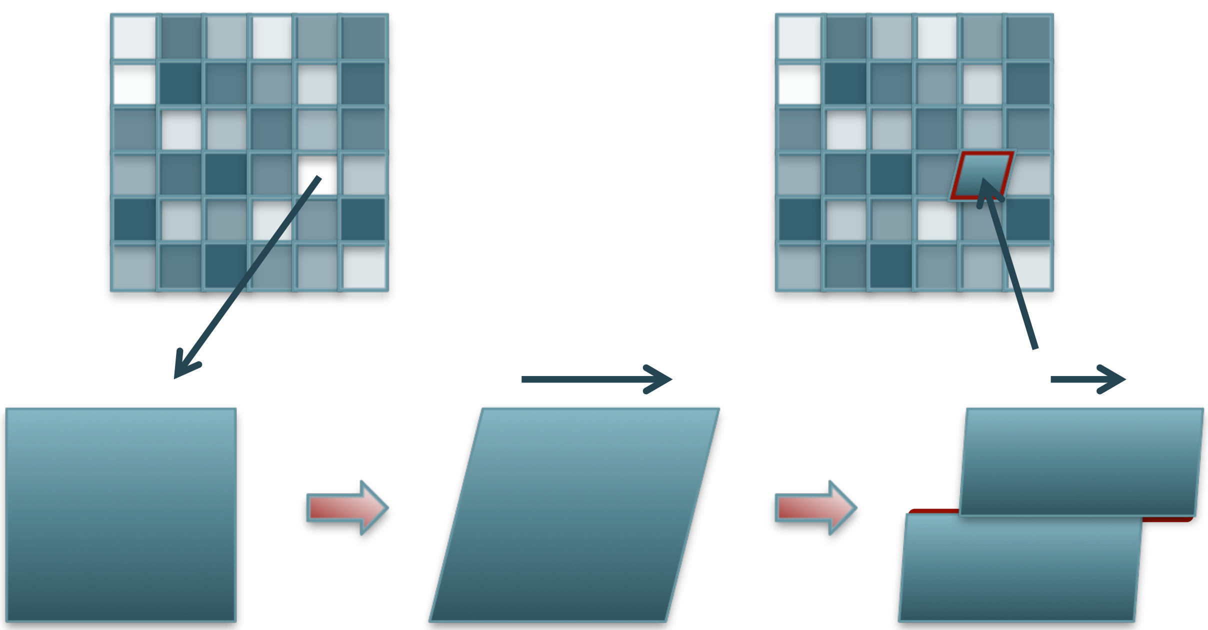
- The microstructure of the material is inhomogeneous
  - Dislocation patterns
  - Random impurities
  - Grain structure
  - The local strength (yield stress) is inhomogeneous in space
- Representing internal disorder
  - Via a local yield threshold
  - RVE size comparable to the scale of disorder
- Deformation will always occur at the ‘weakest’ site

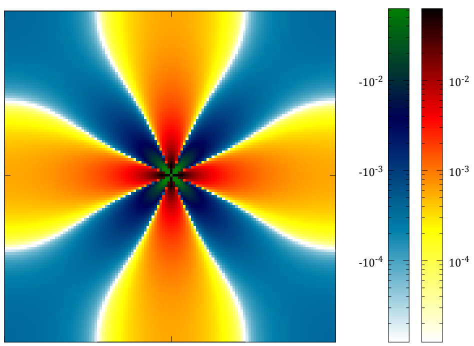


Stochastic plasticity model

Simple local shear deformation:

- Shear stress increases above the local yield threshold
- Plastic deformation: Slip along a single plane
- The local yield threshold changes due to the changed microstructure
- The slipped cell generates $1/r^2$ type elastic stress (Eshelby inclusion problem)



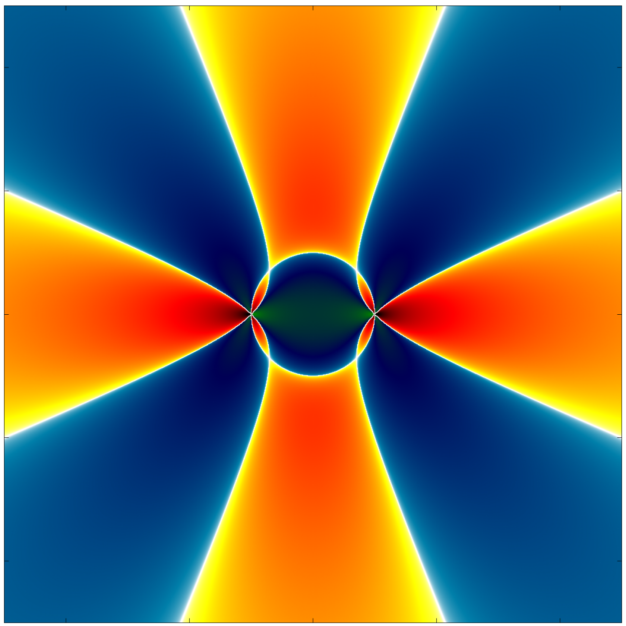


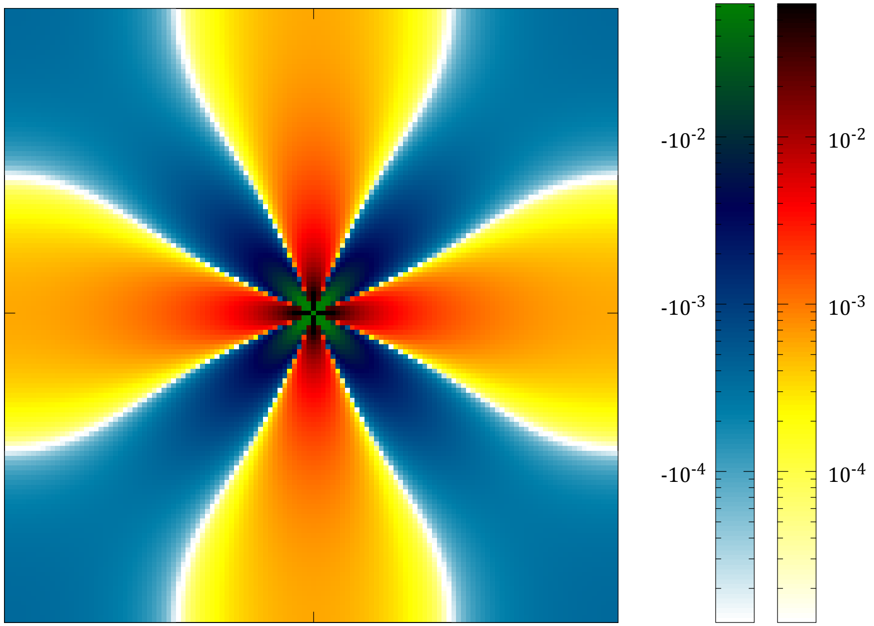
$$\sigma_{xy}(x,y) = \frac{\cos(4\varphi)}{r^2}$$

Zaiser, Moretti, J. Stat. Mech., 2005

Talamali *et al.*, PRE, 2011

Stress field of a dislocation dipole

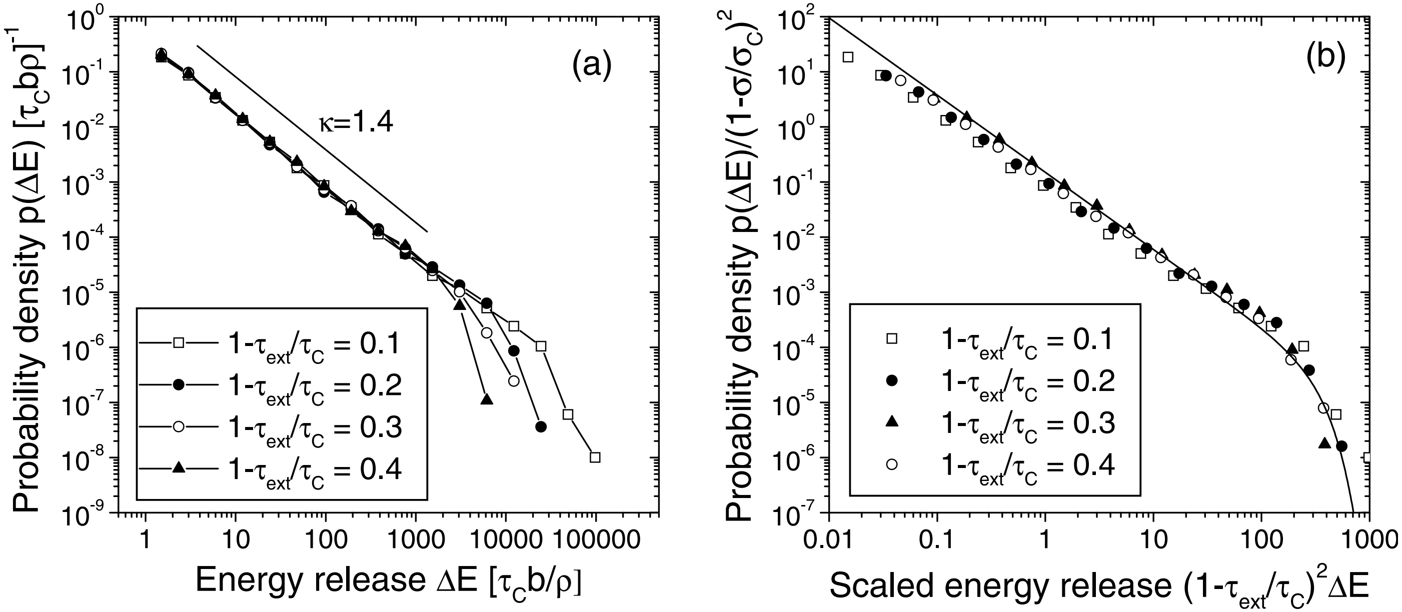




- Shear stress field for large distances: $\sigma_{xy}(x,y) = \dfrac{\cos(2\varphi)}{r^2}$

Depinning in the stochastic model

- Cutoff divergence as external stress (τ_{ext}) tends to the critical threshold (τ_c)

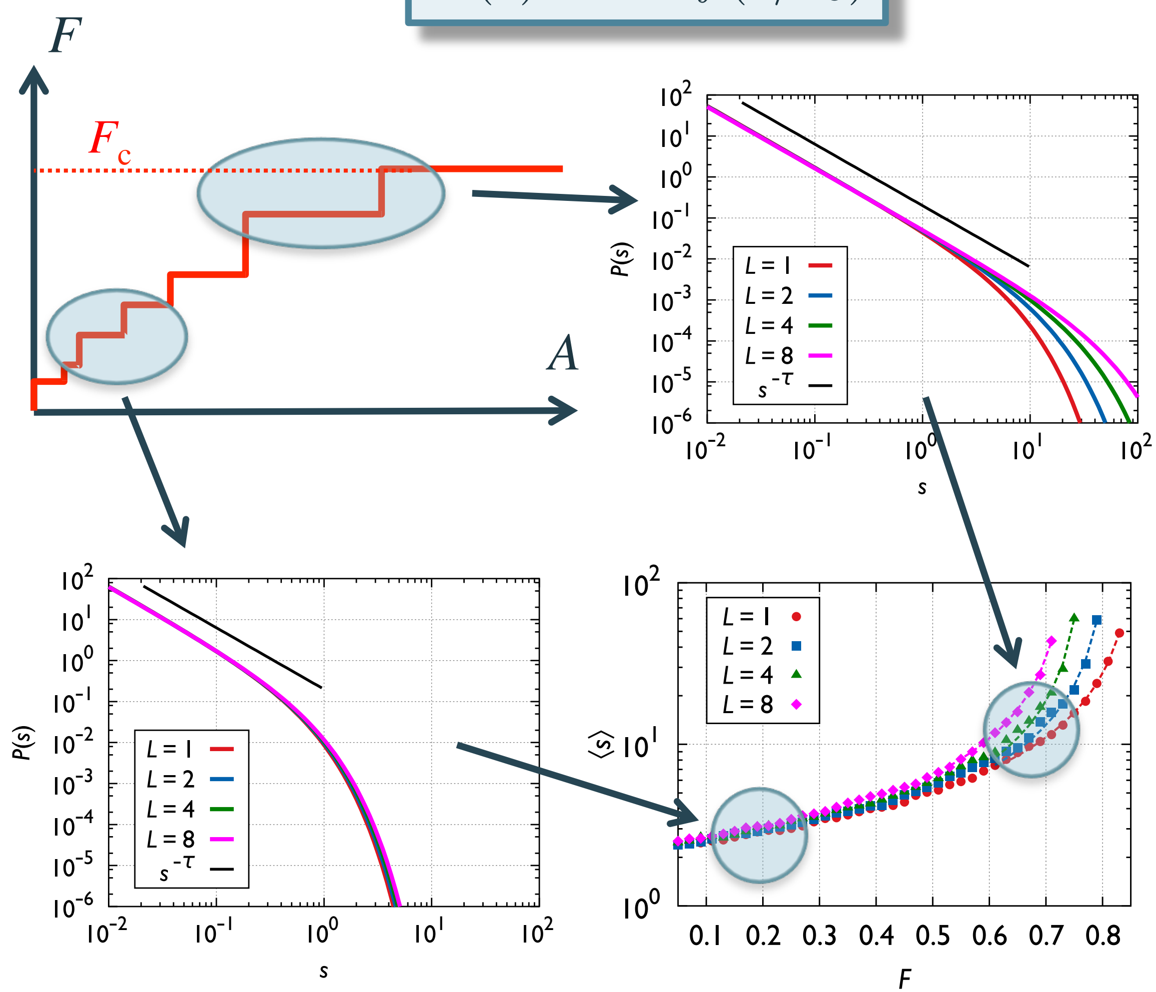


- Dislocation density is constant: no patterns
- Length of RVE? Is there an internal correlation length?

[Zaiser, Moretti, JStat (2005)]

Avalanches during depinning

$$P(s) \propto s^{-\tau} f(s/s_0)$$



- Avalanche size s is the area swept
- The size distribution depends on the force (F) and system size (L)
- The cutoff s_0 diverges at force $F = F_c$:

$$s_0(F) \propto |F - F_c|^{-\alpha}$$

- The average avalanche size $\langle s \rangle$ diverges at F_c
- Finite size effects near F_c:

$$F_c(L) = F_c(\infty) + aL^{-1}$$

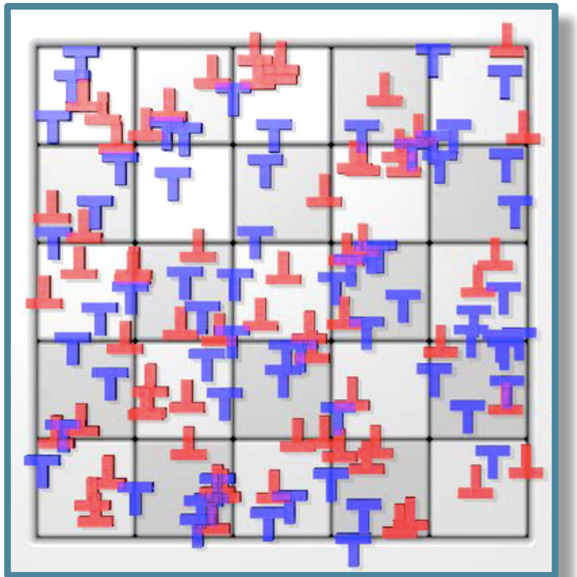
- Mean-field case:
  - $\tau = 1.5$

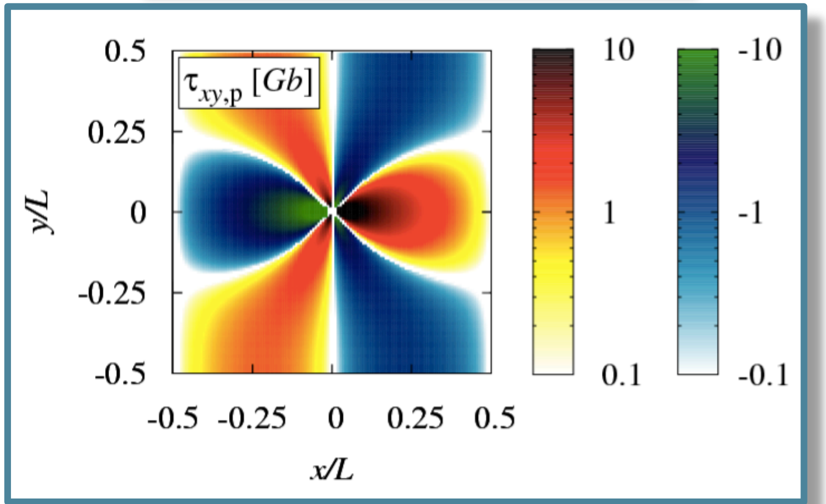
2D dislocation models

2D DDD

- Parallel edge dislocations in single slip
- Periodic boundary conditions
- Number of dislocations: N
- Sign of dislocations: $s_i = \pm 1$
- Stress-field of an individual dislocation: σ_{xy}
- Overdamped motion:

$$\dot{x}_i = s_i \left[\sigma_{\text{ext}} + \sum_{i=1}^{N} s_j \sigma_{xy}(\boldsymbol{r}_i - \boldsymbol{r}_j) \right]$$





Problem formulation

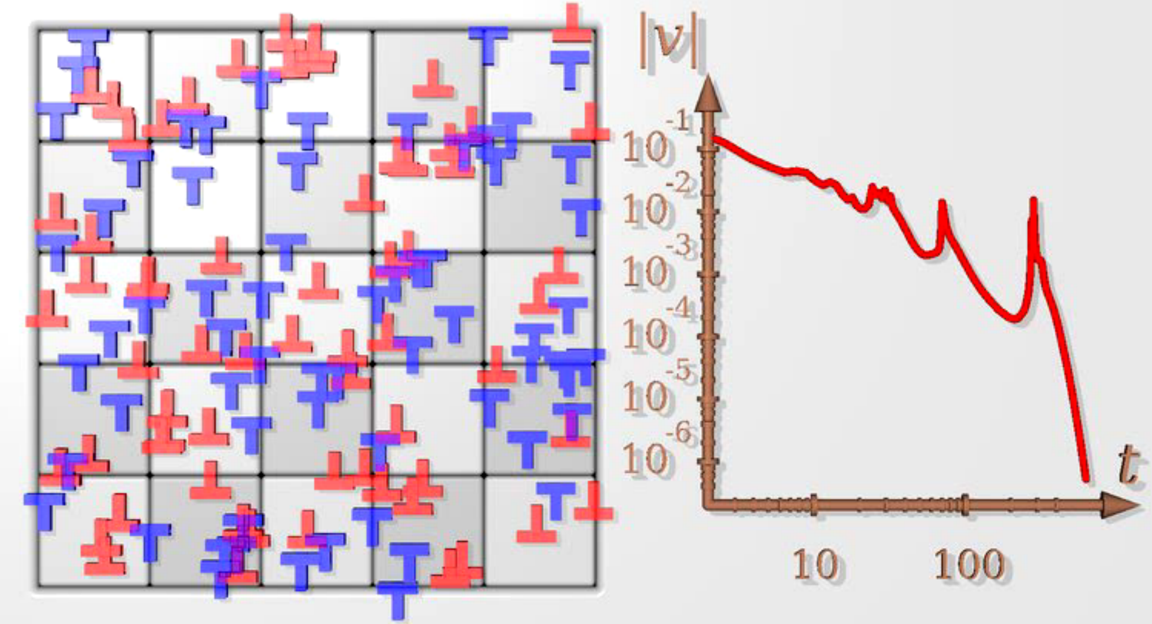
- Energy of a straight edge dislocation: $E = Gb^2 \ln L/r_c$
  - $G = \mu/4\pi(1-\nu)$, L: crystal size, r_c: core radius, b: Burgers vector

- Energy density of *N* randomly positioned straight edge dislocations:
  - The stored energy density is system size dependent
  - The width of the Bragg peaks is also proportional to ln(*L*)

- Dislocation systems cannot be completely random: there must be spatial correlations

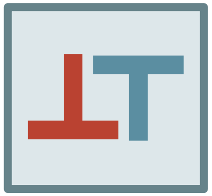
M. Wilkens, 1969

Relaxation from a random initial configuration

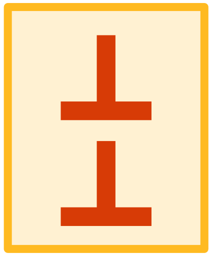


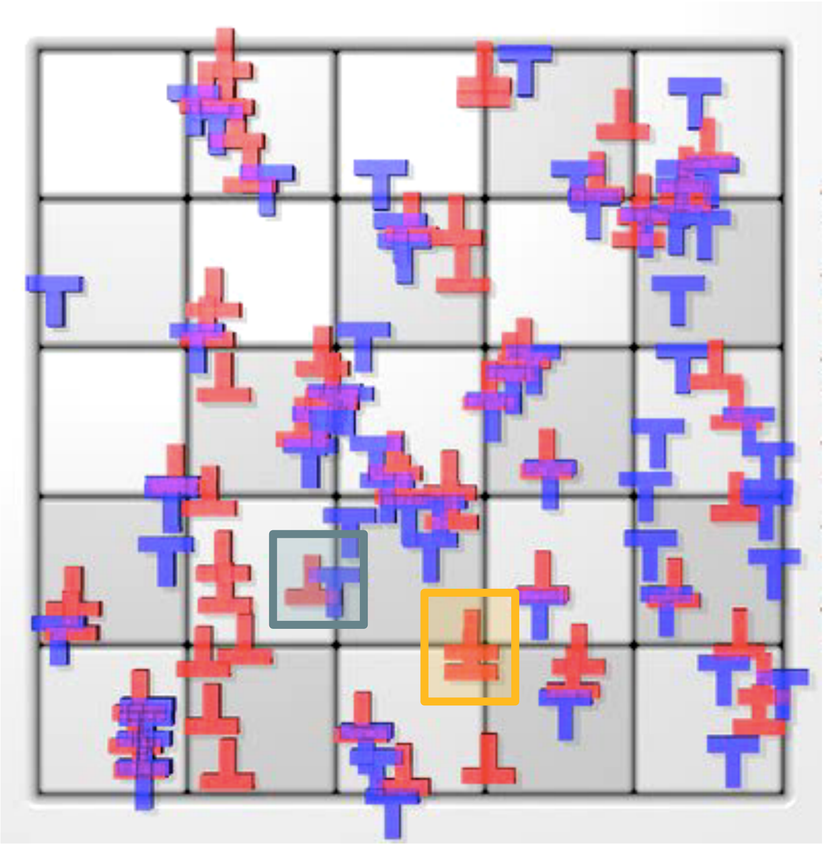
Small-scale configurations

- Equilibrium dislocation configurations are not random: they are characterized by short-range ordering
- Equilibrium positions of a dislocation pair:
  - Opposite sign: *dislocation dipole*



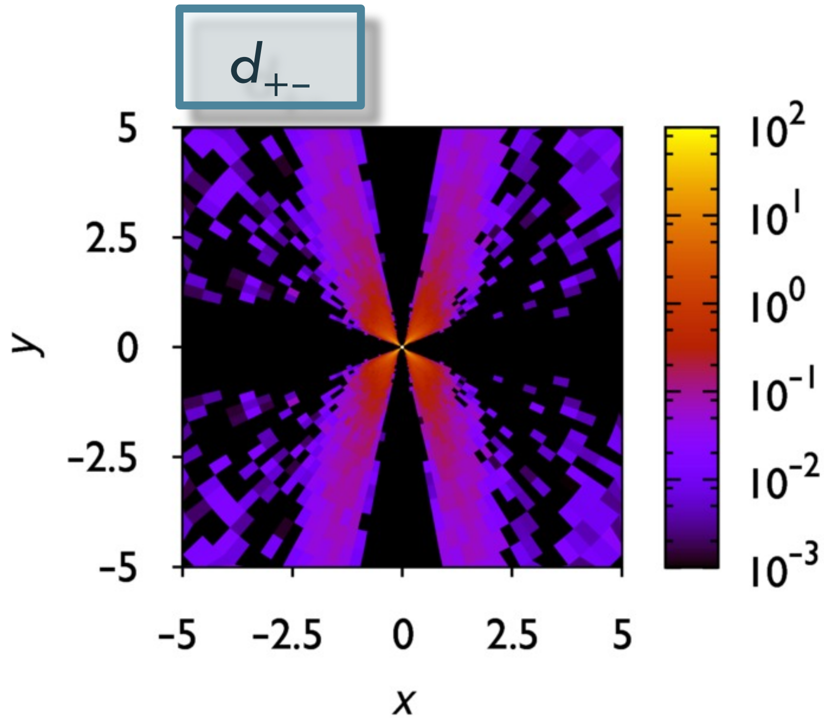
  - Same sign: *dislocation wall*

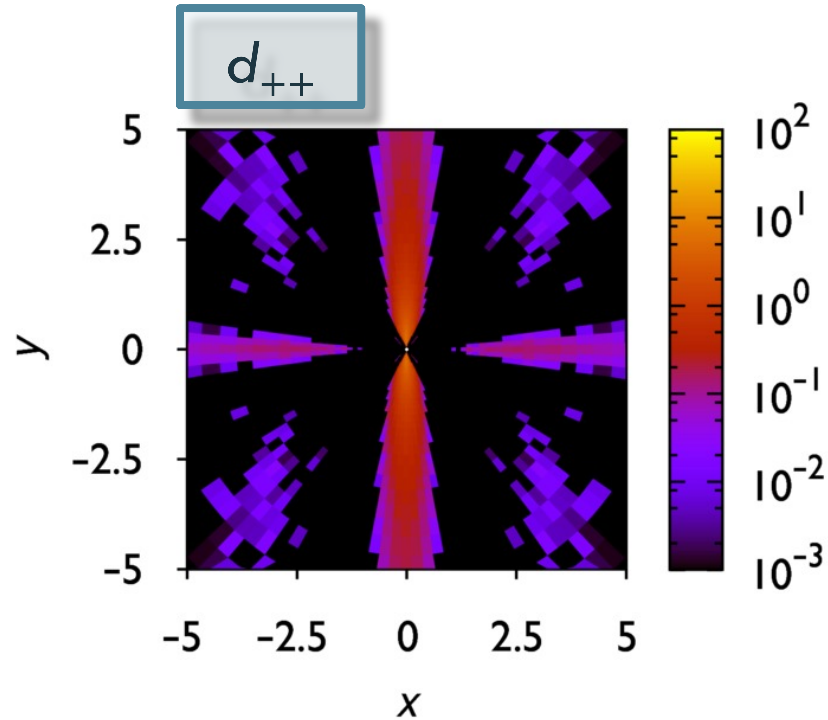




Spatial correlation functions

- Spatial correlation functions: dislcoation density around a randomly chosen dislocation :
  - d_{+-}: relative of the opposite sign dislocations
  - d_{++}: relative of the same sign dislocations, $d_{++}(0,y) \propto |y|^{-1.5}$

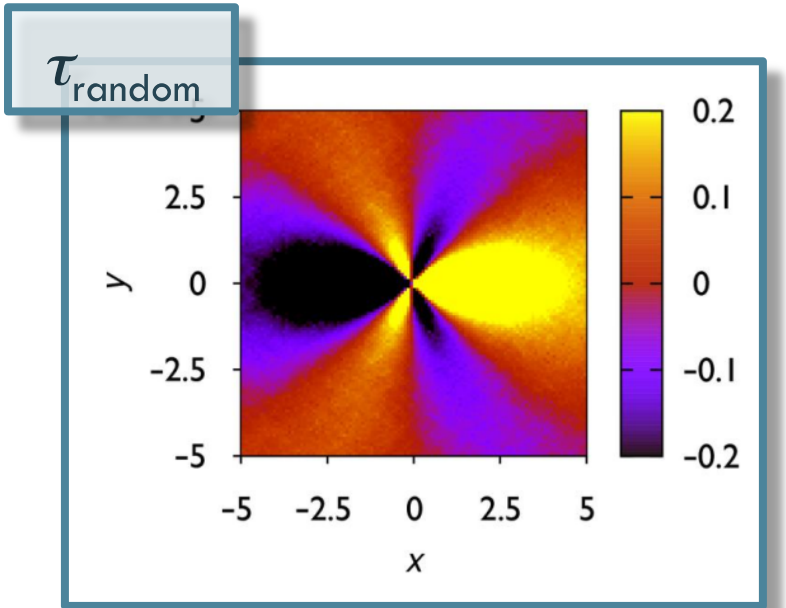


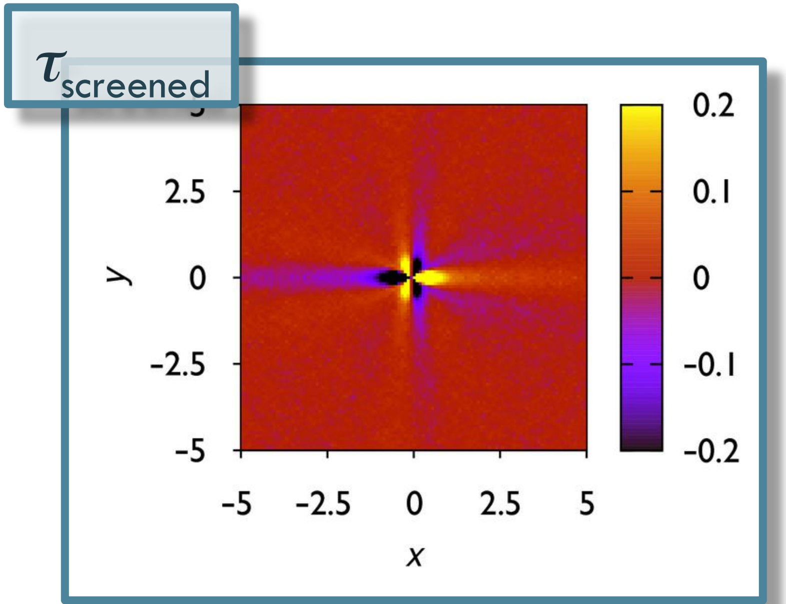


[M. Zaiser *et al.*, PRB, (2002)]

(Debye-)screened stress field

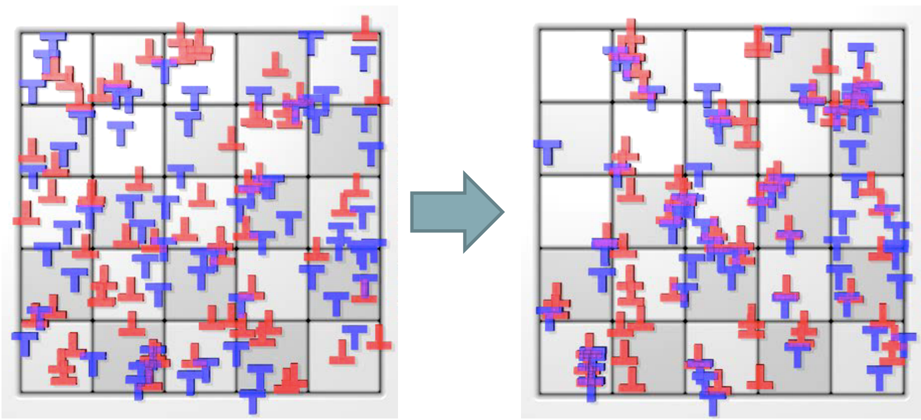
- Elastic energy density of a random dislocation arrangment is superextensive
- Local correlations screen the average stress around each dislocation



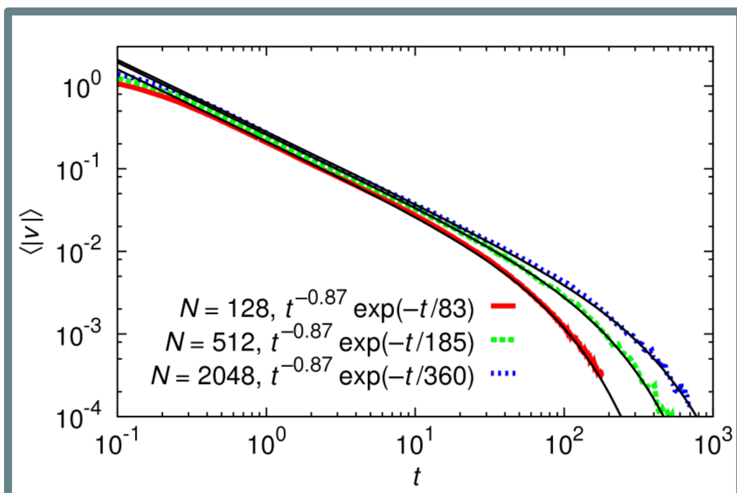


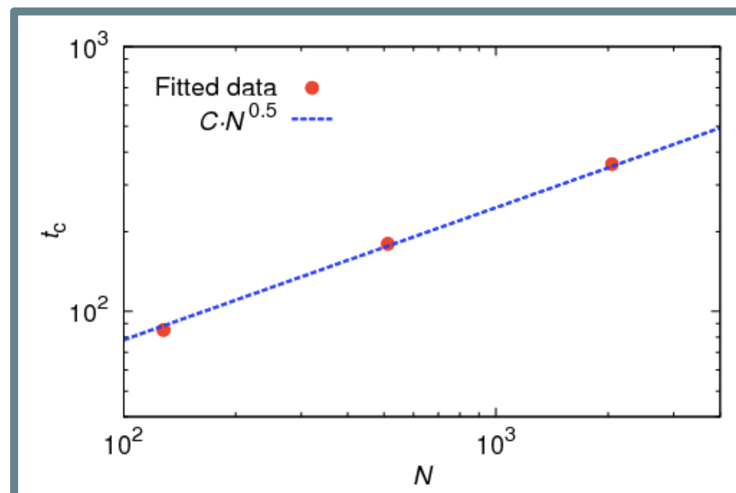
[Groma *et al.*, PRL, (2006)]

Relaxation from a random initial state



- Average absolute velocity: $\langle |v(t)| \rangle \propto t^{-\alpha}, \quad \alpha \approx 0.87$
- The cut-off t_c diverges with system size

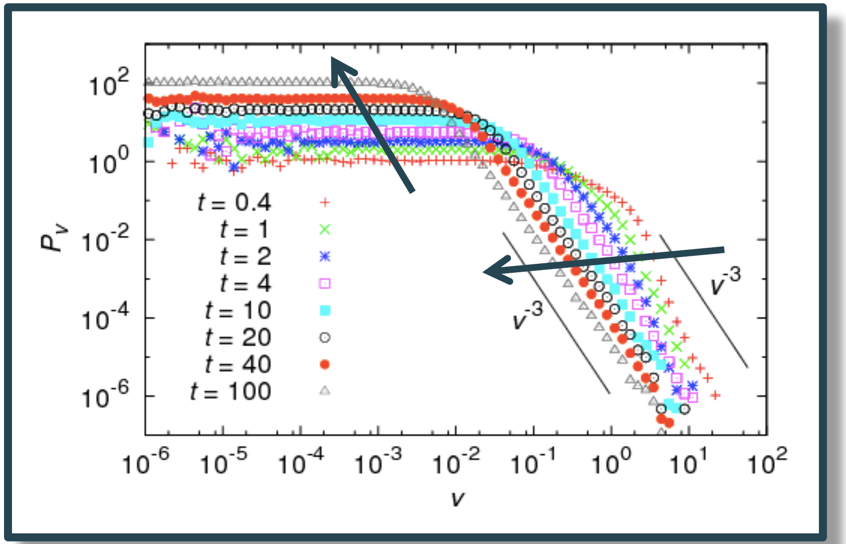


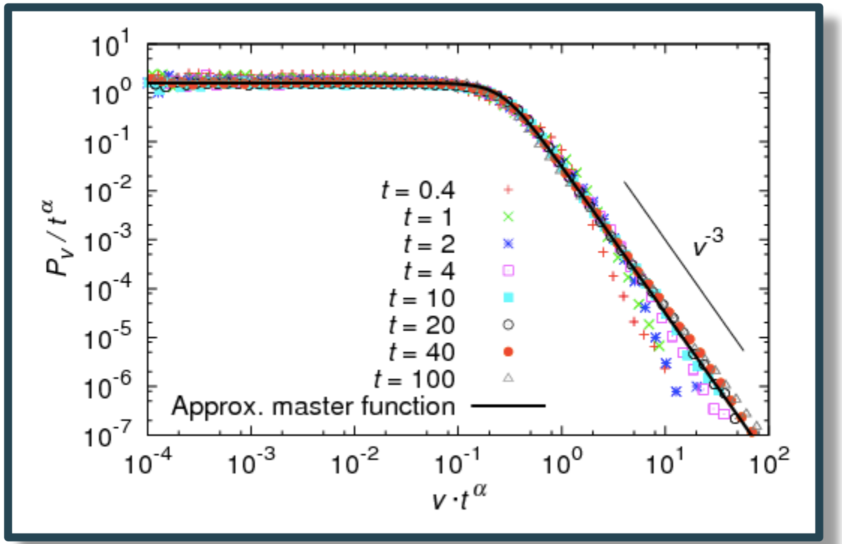


F. F. Csikor *et al.*, JSTAT, 2009; P. D. Ispánovity *et al.*, PRL, 2011

Velocity distribution & scaling

- $P(v,t)$: Distribution of dislocation velocities v at time t
  - Inverse cubic tail (from theory as well)
  - *Tends to a δ-function as t*
- Scaling property: $P(v,t) = t^{\alpha} f(t^{\alpha} v) \quad \Rightarrow \langle |v(t)| \rangle \propto t^{\alpha}$

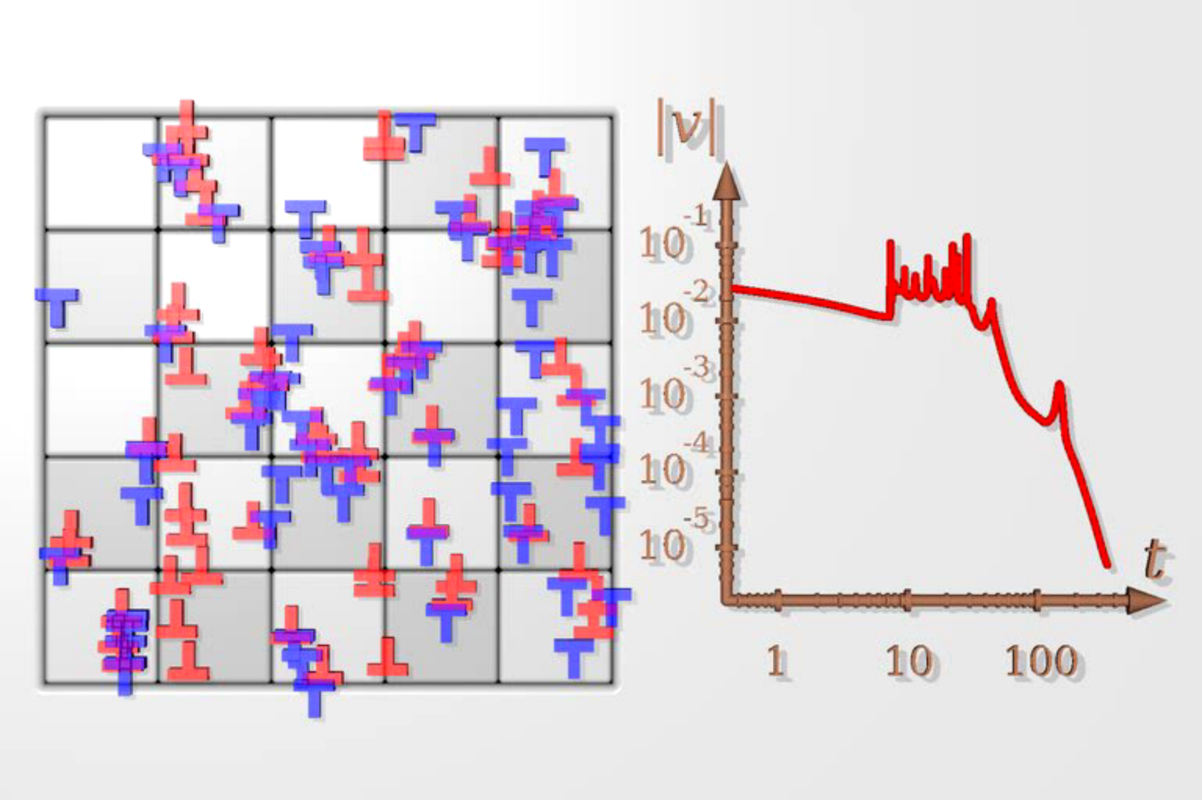


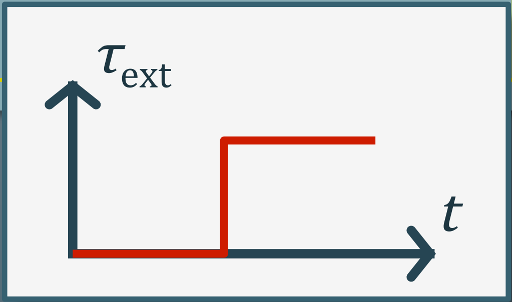


P. D. Ispánovity *et al.*, PRL, 2011

Relaxation due to a small applied stress

- $\tau_{\text{ext}} < \tau_{\text{c}}$
- $N = 128$

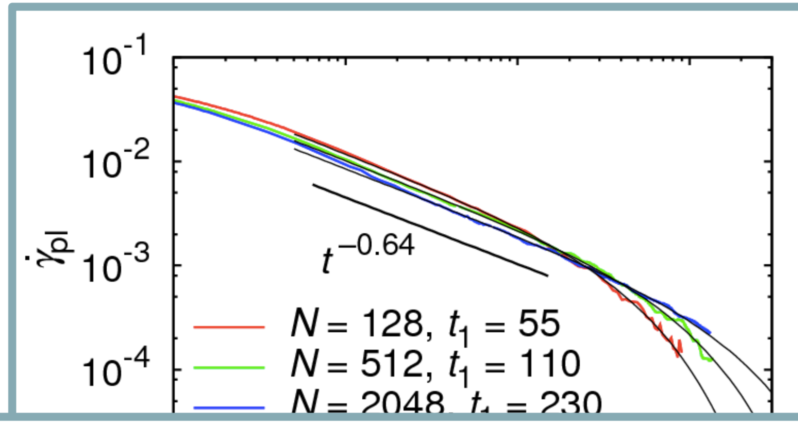




Strain rate evolution

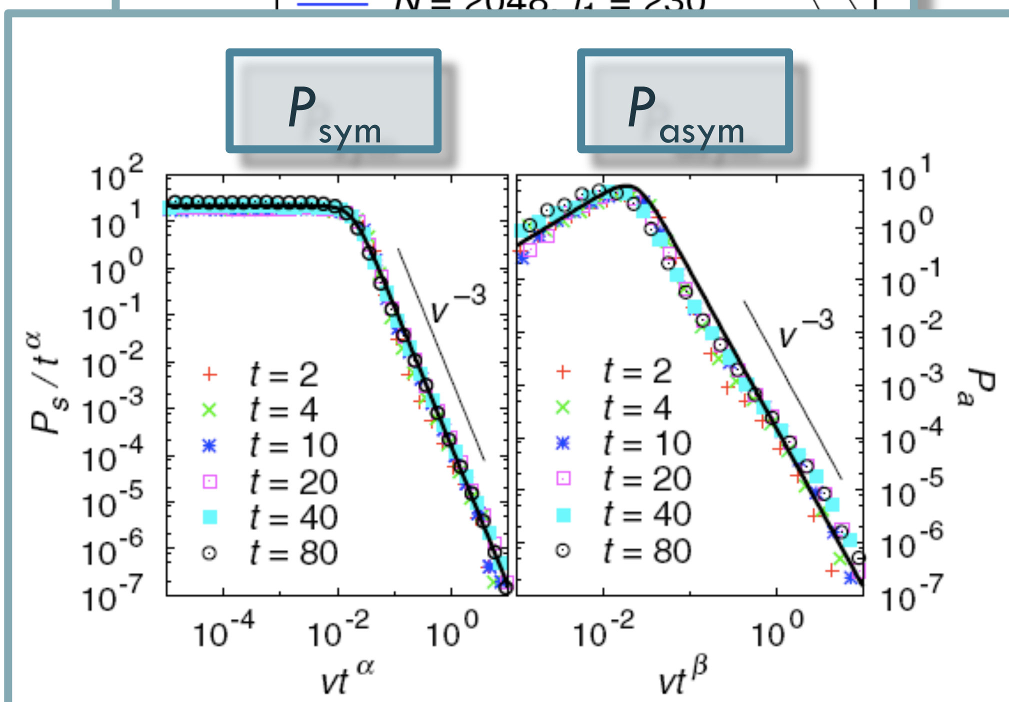
Strain rate:

- $\dot{\gamma}_{\mathrm{pl}} \propto t^{-\delta}$
  (Andrade creep law)
- $\delta \approx 0.64$
- cut-off diverges with sytem size



Velocity distribution:

- Both the symmetric and asymmetric part follows scaling



P. D. Ispánovity *et al.*, PRL, 2011

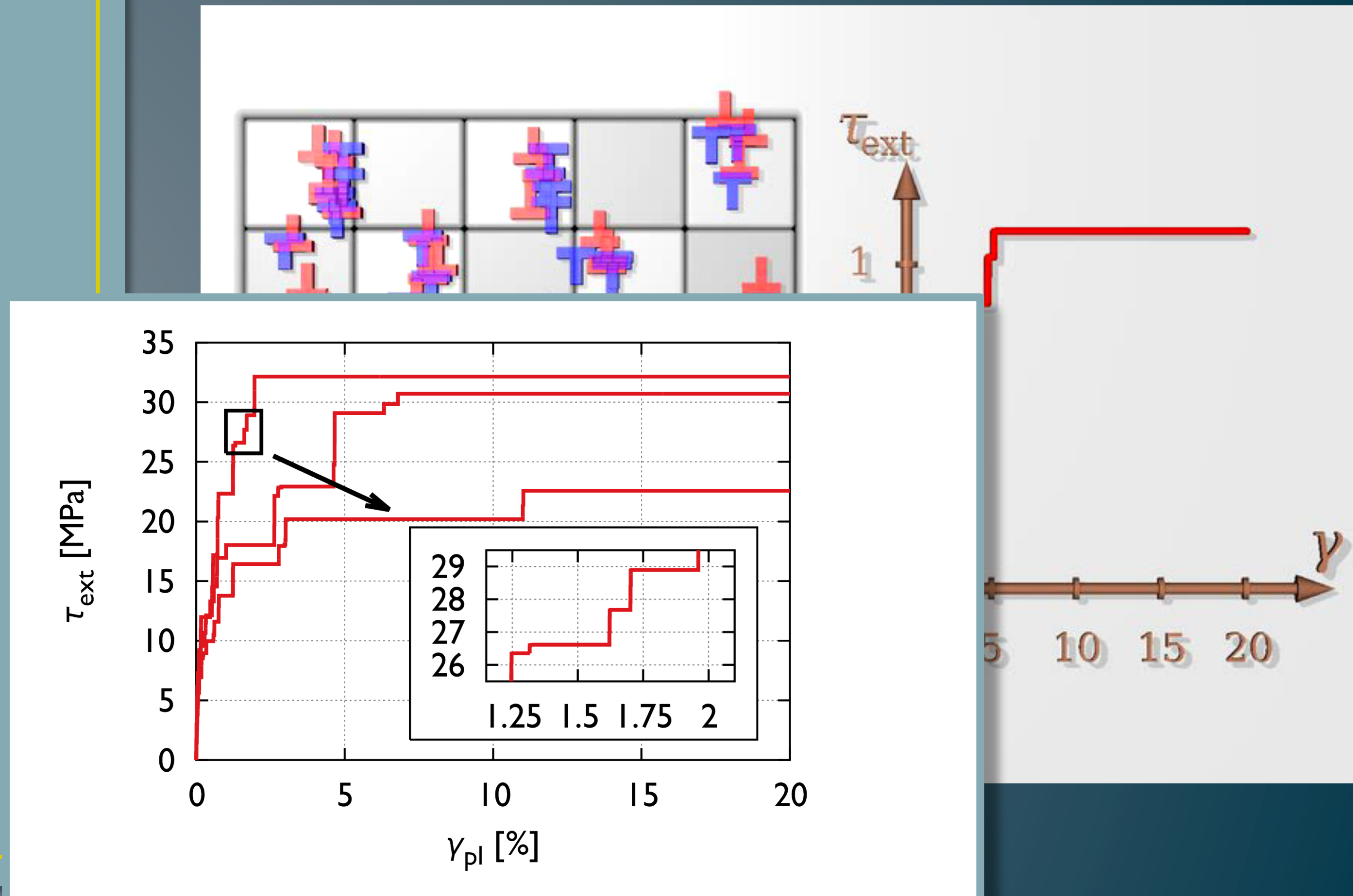
Summary

- Relaxation at different set-ups show slow, power-law relaxation everywhere below the yield stress
- The cut-off time diverges with the system size
- The velocity distributions obey simple scaling relations

↓

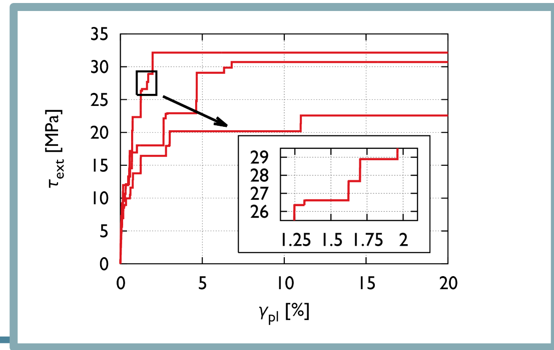
- There is no time scale in the system (except for the scale due to finite system size)
- The system behaves as if it was critical everywhere below the yield stress

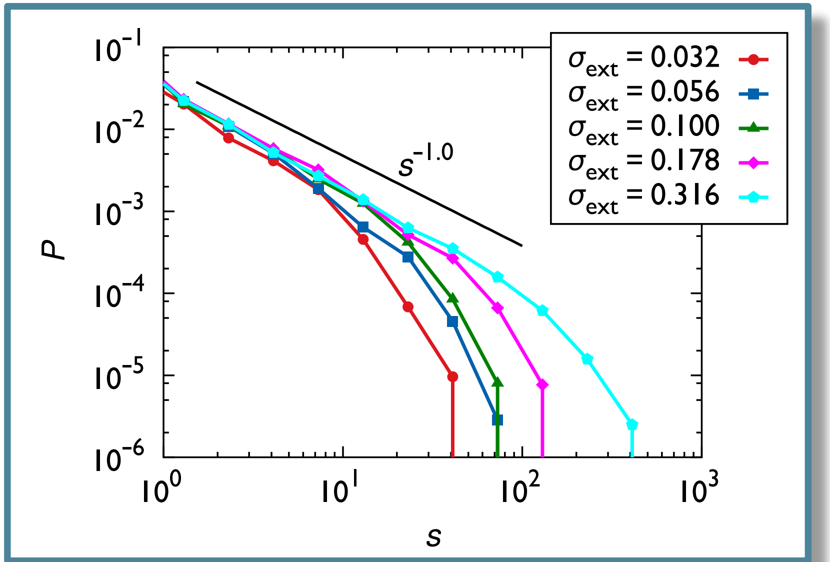
Single slip plastic quasi-static shear in 2D

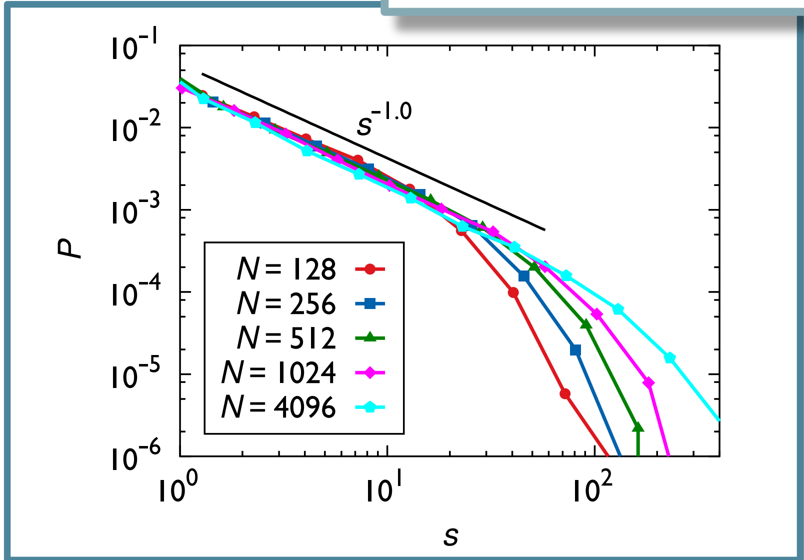


Strain burst distributions

- Avalanche size s is the total area swept
- System size: $L = \sqrt{N}$







Dependence on the applied stress ($N = 4096$)

Dependence on the system size ($\sigma_{ext} = 0.316$)

Ispánovity, Laurson, Zaiser, Groma, Zapperi, Alava, PRL, 2014

Cutoff scaling

Strain burst size distribution:

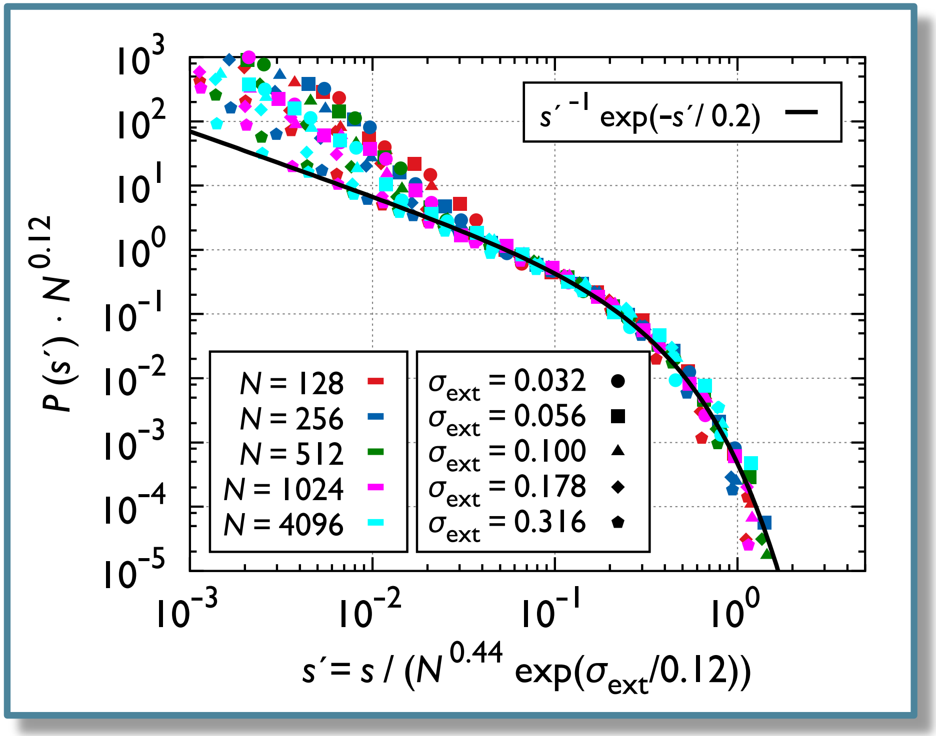
$$P(s) \propto s^{-\tau} \exp(-s/s_0)$$

Cutoff scaling:

$$s_0(\sigma_{\rm ext}, N) \propto N^{\beta} \exp(\sigma_{\rm ext}/\sigma_0)$$

Parameters:

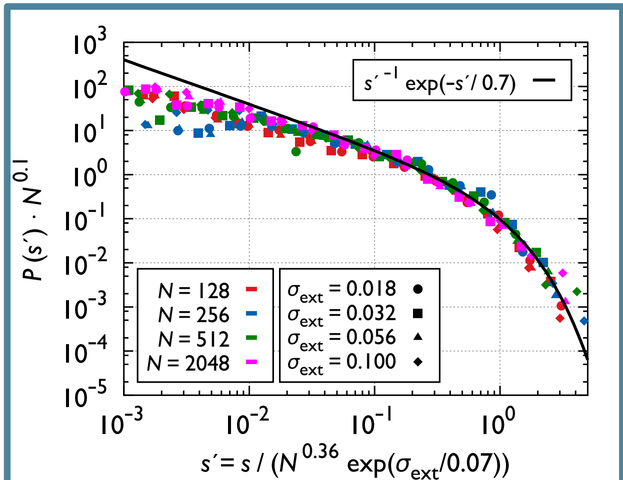
- Avalanche size exp.: $\tau \approx 0.97$
- System size exponent: $\beta \approx 0.36$
- Stress scale: σ_0



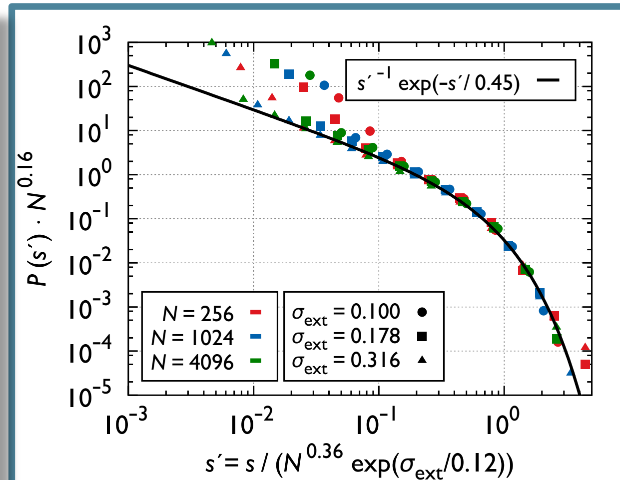
Ispánovity, Laurson, Zaiser, Groma, Zapperi, Alava, PRL, 2014

Comparison of models

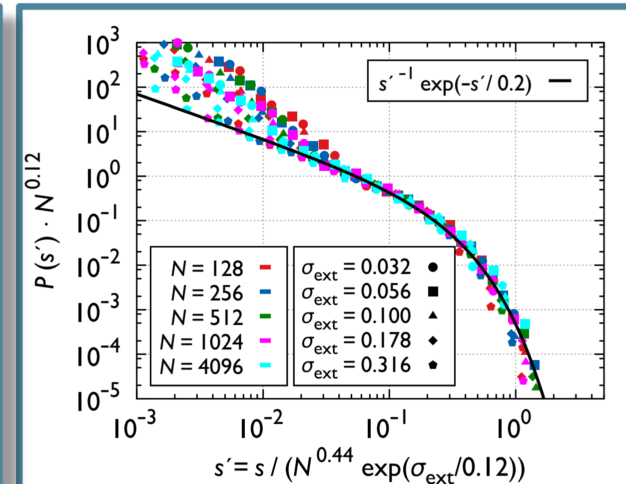
Cutoff behaviour: $s_0(\sigma_{\mathrm{ext}}, N) \propto N^{\beta} \exp(\sigma_{\mathrm{ext}}/\sigma_0)$



Continuous model



CA model with ED



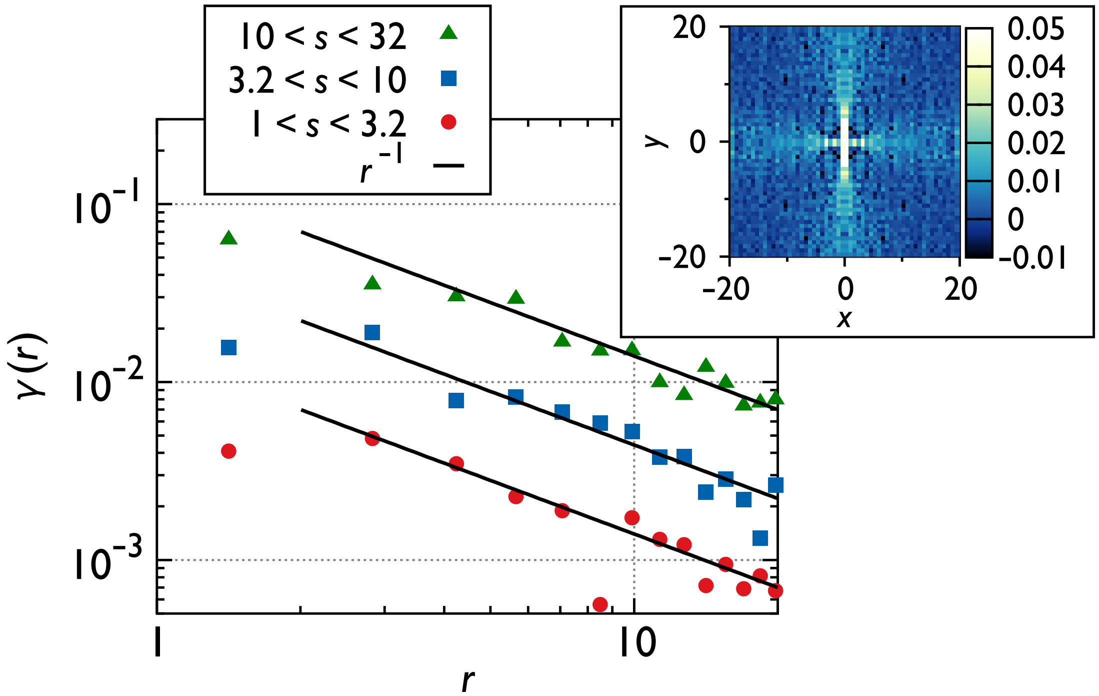
CA model with RD

Model	τ	β	σ_0
Continuous	0.97±0.03	0.36±0.04	0.07±0.01
CA with ED	1.00±0.03	0.36±0.02	0.116±0.004
CA with RD	1.02±0.01	0.44±0.01	0.122±0.002

- $\tau \approx 1.0$
- $\beta \approx 0.4$

Average avalanche shape

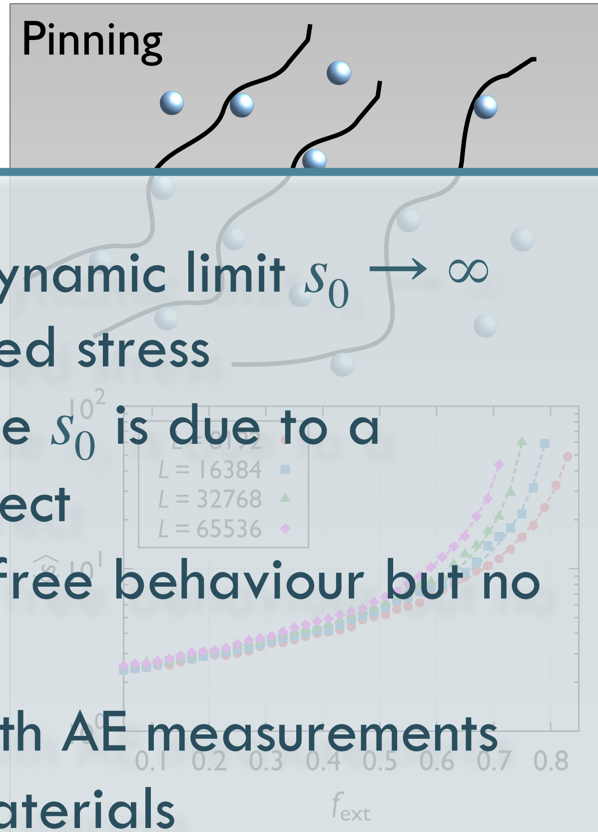
- Avalanche shape following the Eshelby directions
- $1/r$ decay
- No cutoff in the avalanche shape



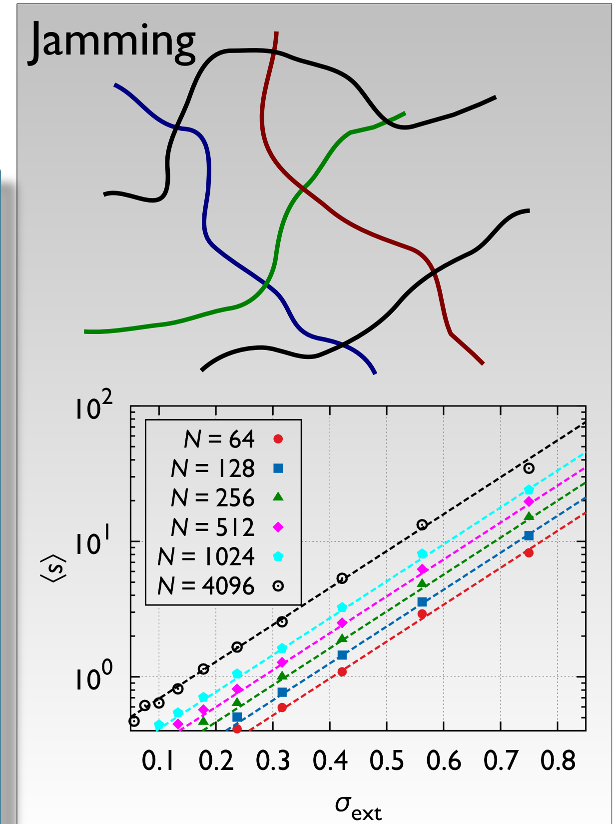
Ispánovity, Laurson, Zaiser, Groma, Zapperi, Alava, PRL, 2014

Non-depinning behaviour

- In the thermodynamic limit $s_0 \to \infty$ at every applied stress
- The length scale s_0 is due to a system size effect
- There is scale-free behaviour but no critical point
- Compatible with AE measurements on pure hcp materials



$$\langle s \rangle \propto (\sigma_c(L) - \sigma_{ext})^{-\alpha}$$

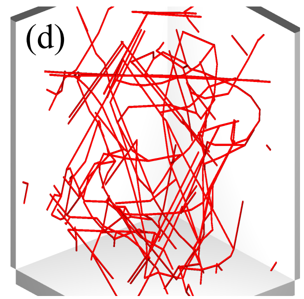


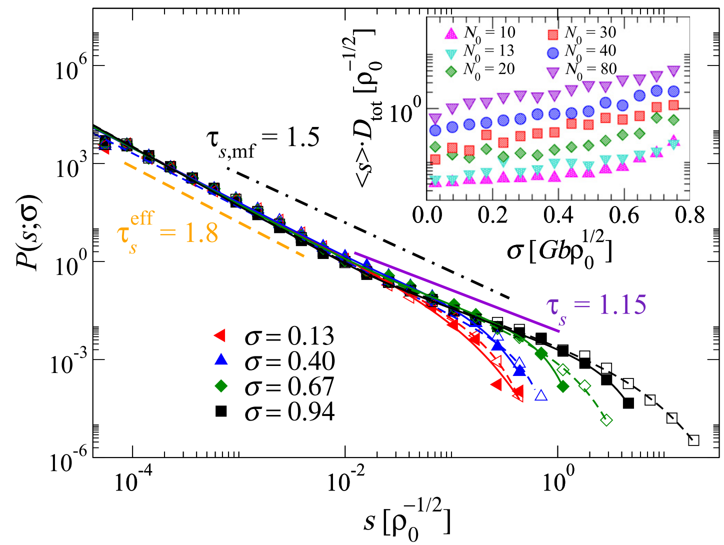
$$\langle s \rangle \propto L^{2\beta} \exp(\sigma_{ext}/\sigma_0)$$

$$\langle \gamma \rangle = L^{2\beta-2} \exp(\sigma_{ext} / \sigma_0)$$

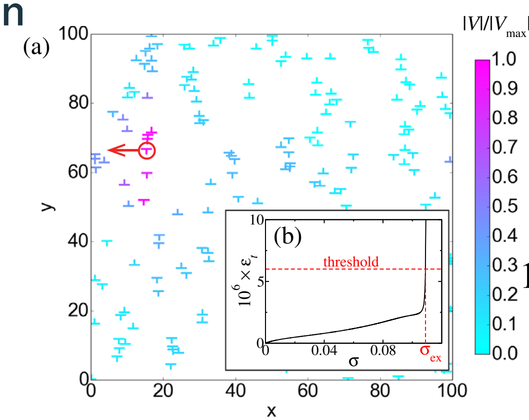
Further evidence

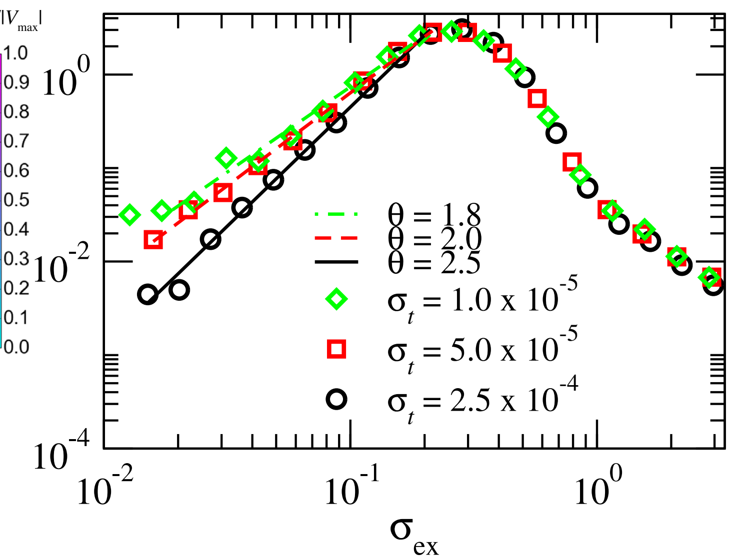
- Glassy avalanche distributions in 3D DDD simulations





- Power-law distributions for the local yield stress (in 2D and 3D)
  - Only one dislocation or a local region is excited
  - The exponent θ dependes on loading rate



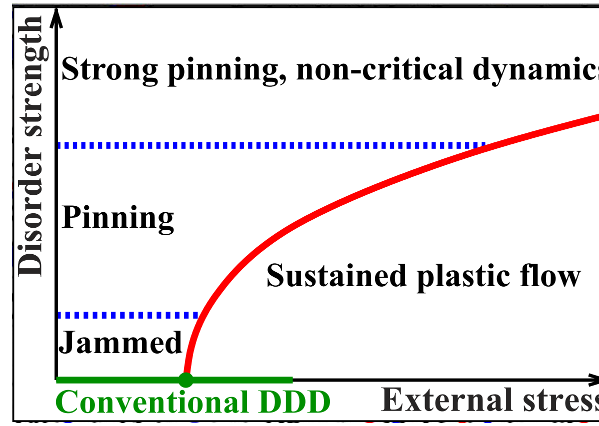


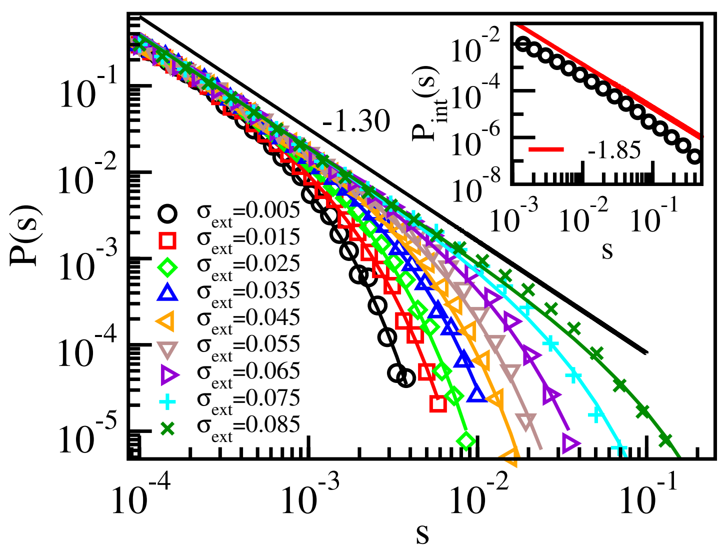
[Lehtinen, Costantini, Alava, Zapperi, Laurson, PRB (2016)]

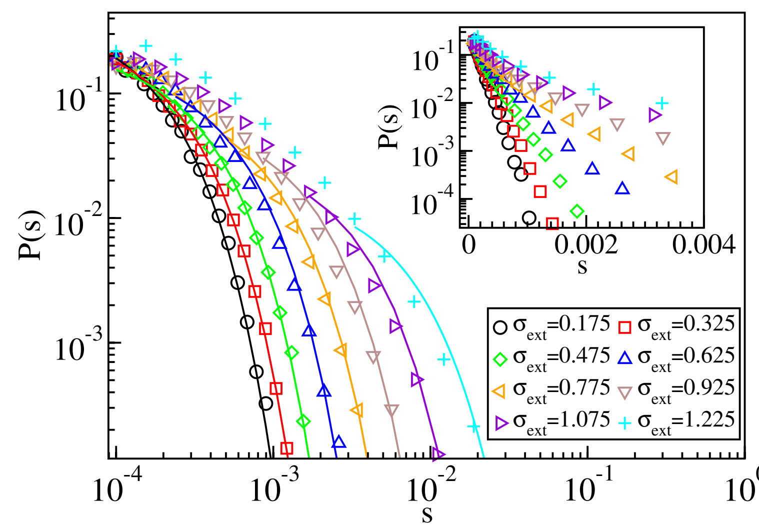
[Ovaska, Lehtinen, Alava, Laurson, Zapperi, PRL (2017)]

Effect of quenched pinning

- A new length-scale leads to:
  - jamming → pinning → Gaussian dynamics
  - A gap in the local threshold distribution







[Ovaska, Laurson, Alava, Sci. Rep. (2015)]

Summary

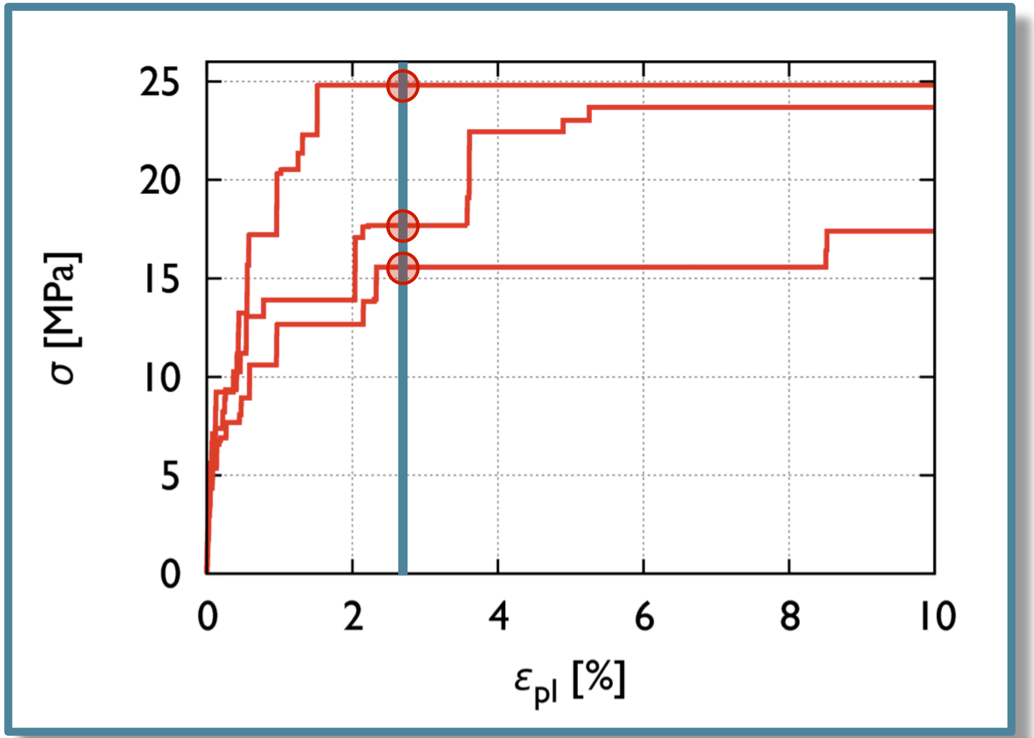
- Pure dislocation systems exhibit extended criticality
  - There is no correlation length associated with dislocation structure
  - Slow relaxation
  - The avalanche size diverges at every applied stress level
- Plastic yielding in general cannot be described in terms of a depinning transition
- Robust: recovered in 3D simulations as well
- This picture may be compatible with experiments on pure hcp or fcc materials
- Introduction of internal length scale changes the picture
  - Depinning or Gaussian fluctuations

Outlook

- Questions
  - How does the transition from jamming to depinning happen?
    - System size dependence
    - Is it possible to get into extended crytical region by loading?
  - What influences the exponent?
  - How does strain hardening and other phenomena affect universality?
  - How to go to higher scales? What is the RVE?

- Work in progress
  - Implementing finite boundaries
  - New algorithms to allow for larger systems/less numerical noise
  - Get closer to experiments: multiple slip, Peierls stress
  - Local yield threshold distributions

Statistical analysis of stress-strain curves

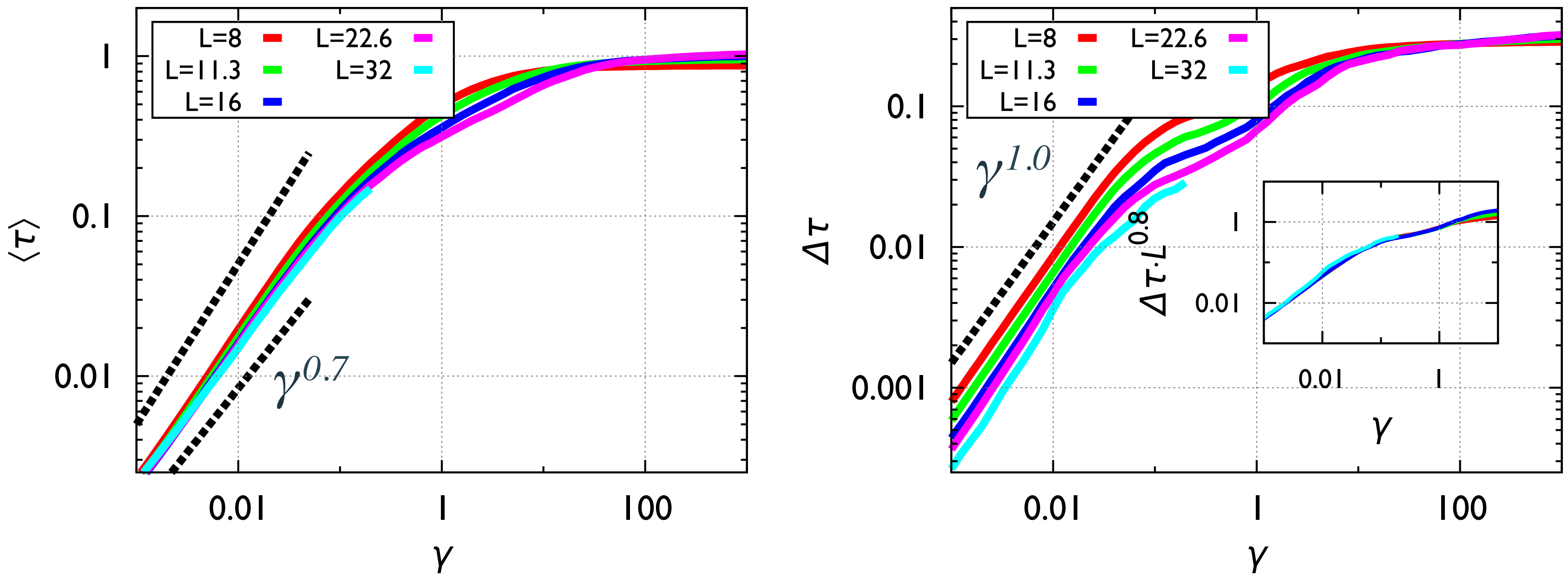


- Stress values for a given plastic strain level
- Distribution of these stress values

Average stress-strain curves

- Average and standard deviation of stress strain curves

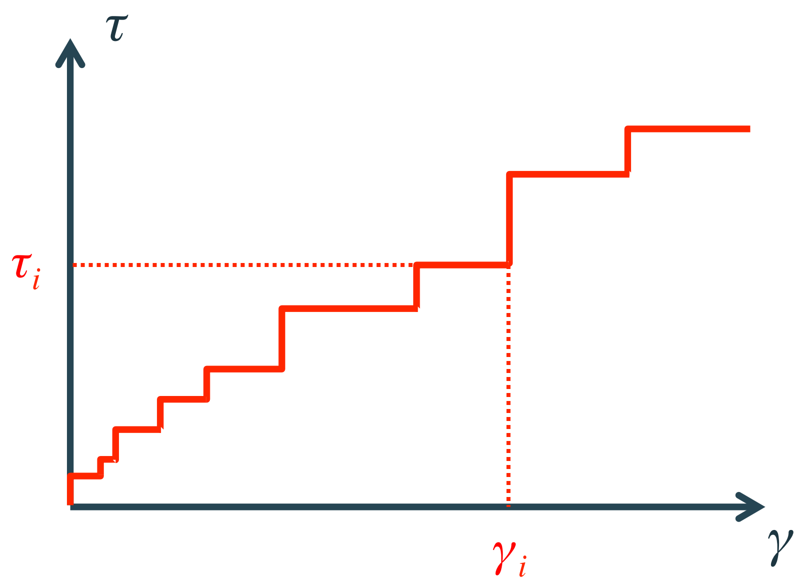
$$\langle \tau \rangle \propto \gamma^{0.7} \qquad \Delta\tau \propto \gamma^{1.0} / L^{0.8}$$



- There is only a negligible size effect!

Stress sequence statistics

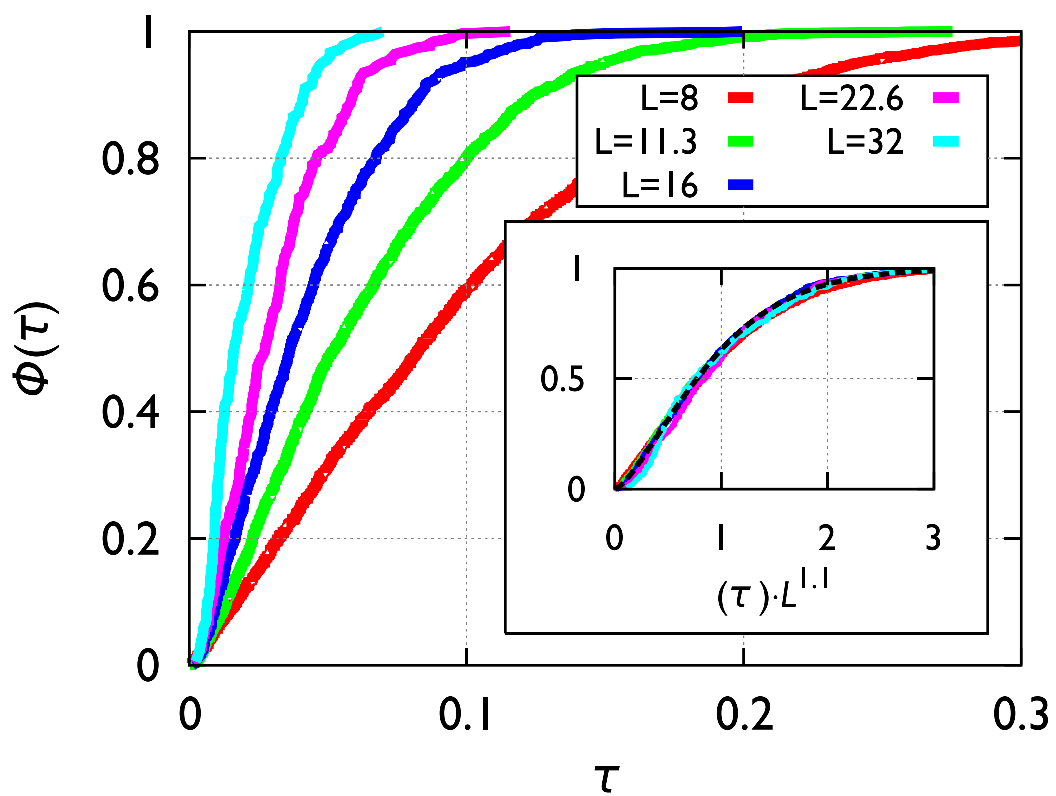
- The stress-strain curve is a sequence of discrete stress and strain values:
  - τ_i: shear stress at the ith event
  - γ_i: shear strain at the ith event



- Distribution of strain increments $\Delta\gamma_i = \gamma_i - \gamma_{i-1}$ has been discussed

- What is the distribution of τ_i?

Distribution of τ_1



- For larger systems stress values get smaller:

$$\left\langle \tau^{(1)} \right\rangle \propto 1/L$$

- Cumulative distribution is a Weibull:

$$\Phi^{(1)}(\tau_1) = 1 - \exp\left(-\left(\frac{\tau_1}{\lambda} \right)^{\nu} \right)$$

- Shape parameter: $\nu \approx 1.4$

Background of the Weibull distribution

- A chain consists of independent links
- If the asymptotic ($\tau \to 0$) failure distribution of **one link** is:

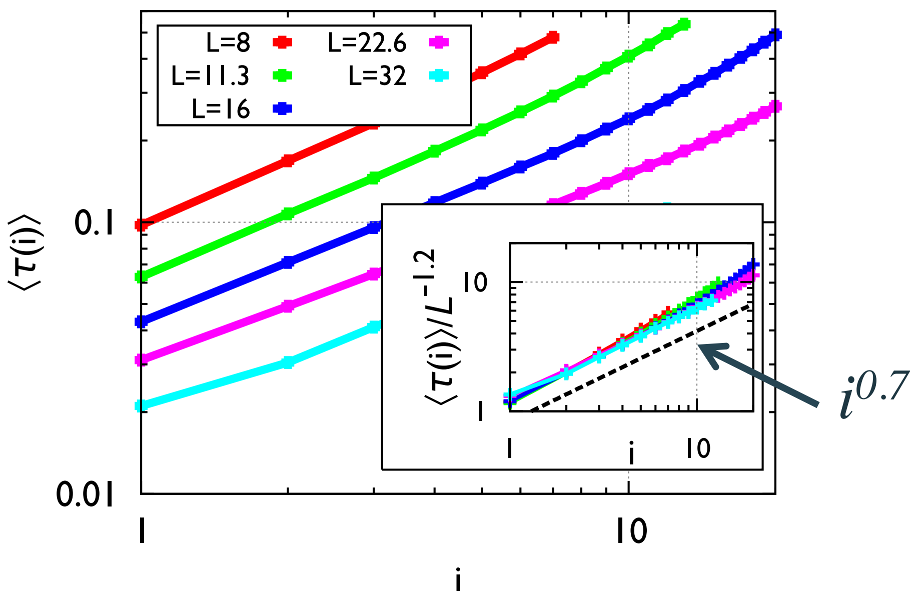
$$\Phi(\tau) = \left(\frac{\tau}{\tau_0}\right)^{\nu} \quad \Rightarrow \quad P(\tau) = \Phi'(\tau) = \frac{\nu}{\tau_0}\left(\frac{\tau}{\tau_0}\right)^{\nu-1}$$

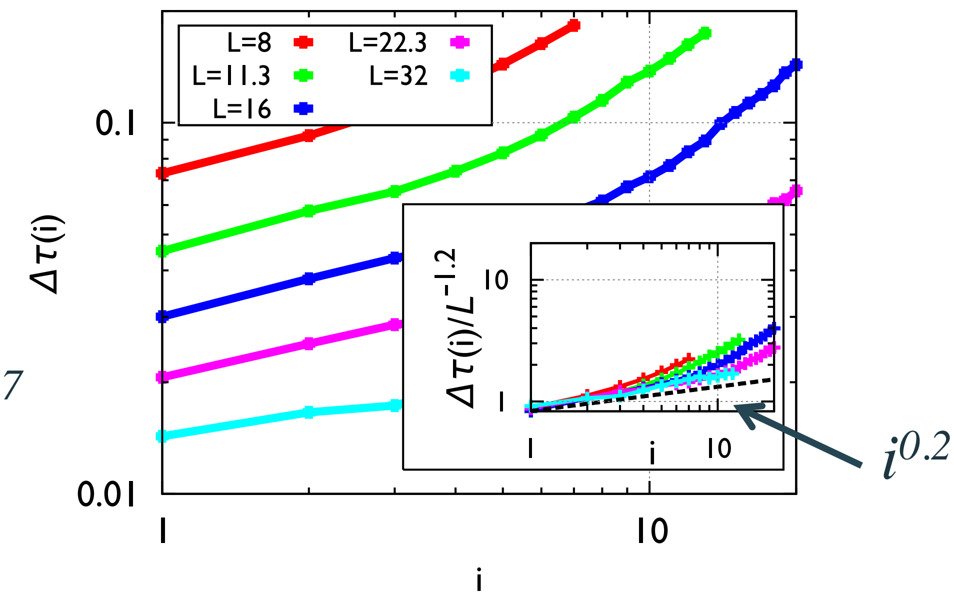
- then the failure distribution of the **chain of *M* links** is

$$\Phi^{(1)}(\tau^{(1)}) = 1 - \exp\left(-\frac{1}{M}\left(\frac{\tau^{(1)}}{\tau_0}\right)^{\nu}\right)$$

Stress sequence

- What about the statistics of τ_i?
- Mean: $$\langle \tau^{(i)} \rangle = \frac{\tau_0}{M^{1/\nu}} \frac{\Gamma(i+1/\nu)}{\Gamma(i)} \approx \tau_0 \left(\frac{i}{M}\right)^{1/\nu}, \quad \text{if } i \geq 5$$
- Standard deviation: $$\Delta\tau^{(i)} \approx \frac{\tau_0}{i^{1/2}} \left(\frac{i}{M}\right)^{1/\nu}, \quad \text{if } i \geq 5$$



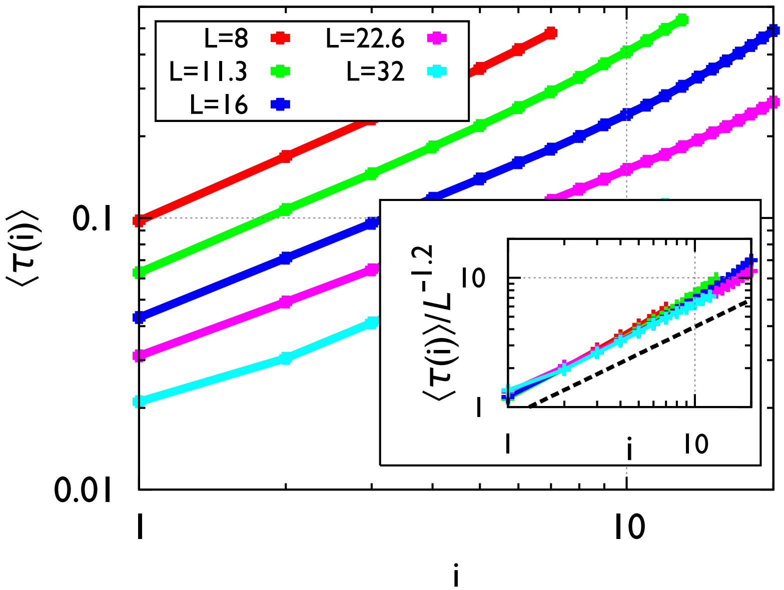


[Derlet, Maas, Philos. Mag. 2015]

Size dependence

- How does the number of links *M* depend on the system size?
- Assumption: $M \propto L^{\xi}$
  - If links are homogeneously distributed, then $\xi = 2$.

$$\left\langle \tau^{(i)} \right\rangle \approx \tau_0 \left(\frac{i}{M} \right)^{1/\nu} = \frac{\tau_0}{L^{\xi/\nu}} i^{1/\nu}$$

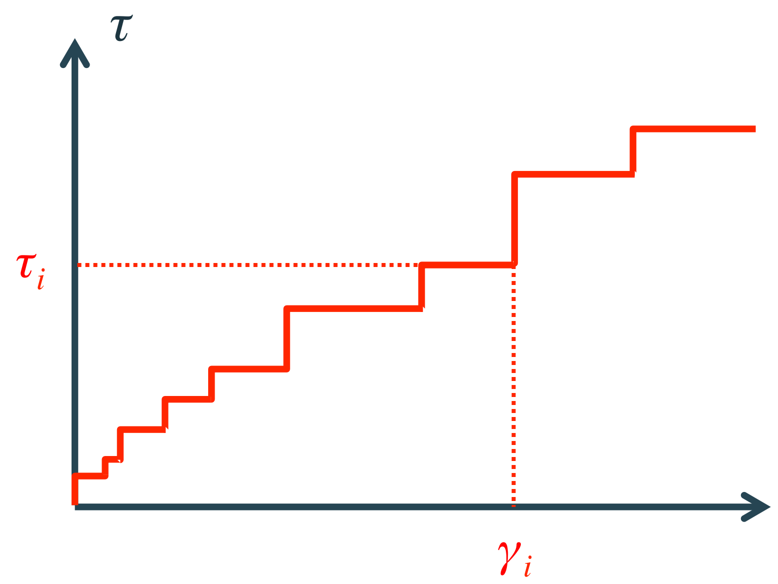


  - From the fit $\xi / \nu \approx 1.2 \Rightarrow \xi = 1.2 \cdot \nu \approx 1.7$
- The numer of links grows subextensively with the system size

Plasticity model

- Stresses τ_i are an independent random sequence drawn from a Weibull distribution with shape parameter υ.
- Strain increments are independent random variables with finite moments:

$$\left\langle \gamma^{(i)} \right\rangle = \frac{s_0}{L^\eta} i$$



- Average stress-strain curve: $\left\langle \tau \right\rangle \propto \left(\frac{L^\eta}{L^\xi} \right) \left\langle \gamma \right\rangle$

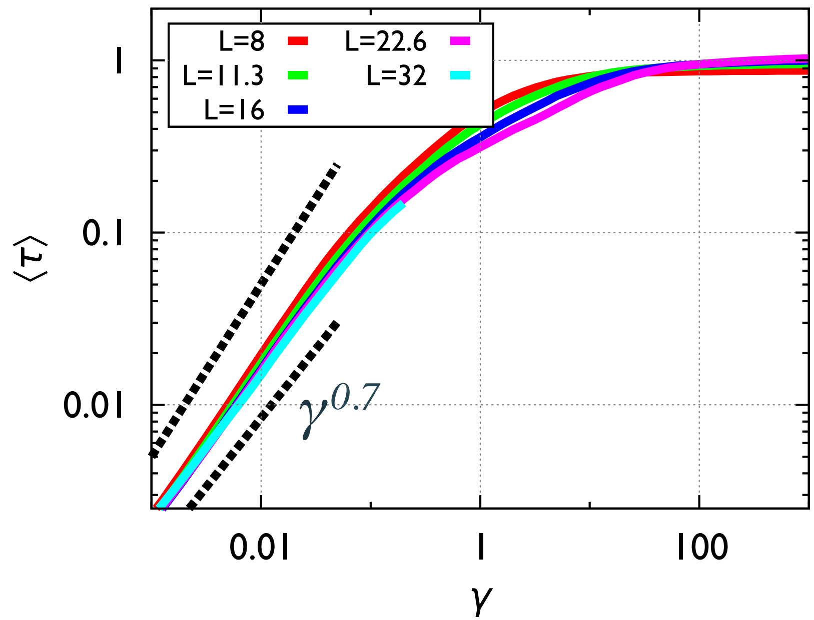
- Standard deviation of stresses: $\Delta\tau \propto L^{-\xi/2} \left\langle \gamma \right\rangle$

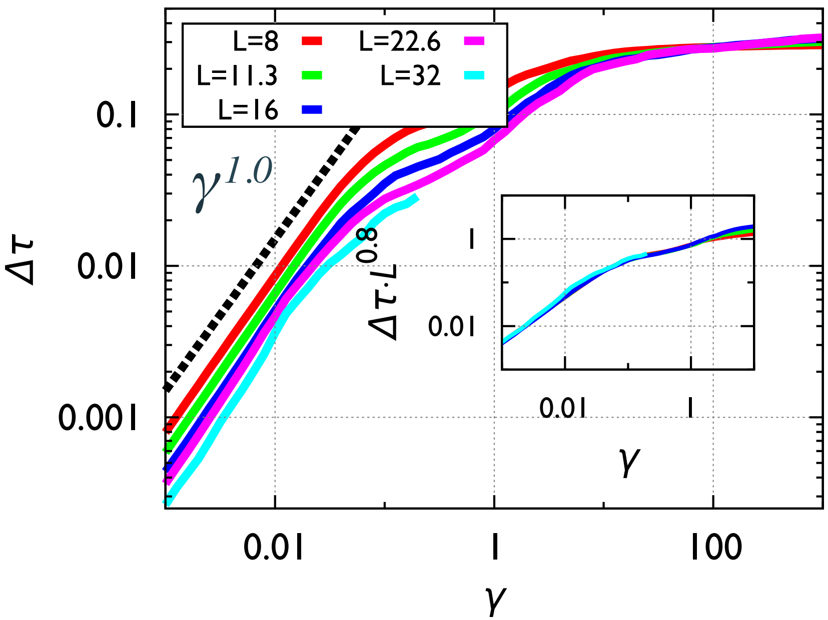
Average stress-strain curves

- Average and standard deviation of stress strain curves

$$\langle \tau \rangle \propto \gamma^{0.7}$$

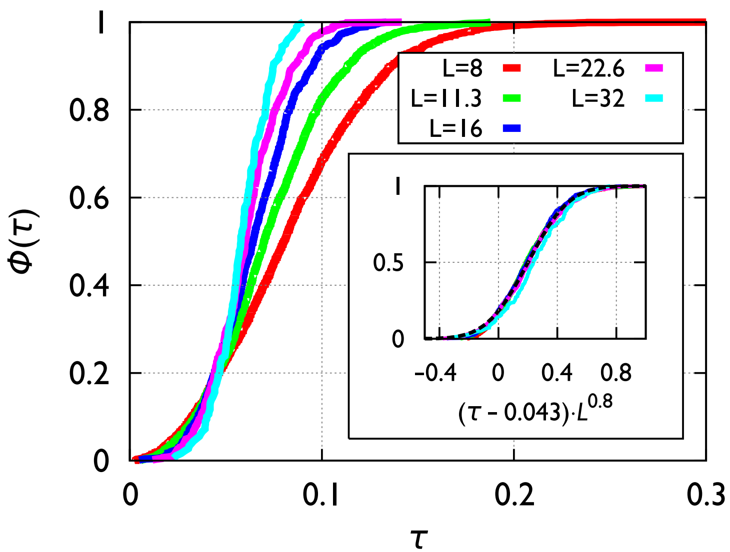
$$\Delta\tau \propto \gamma^{1.0} / L^{0.8}$$



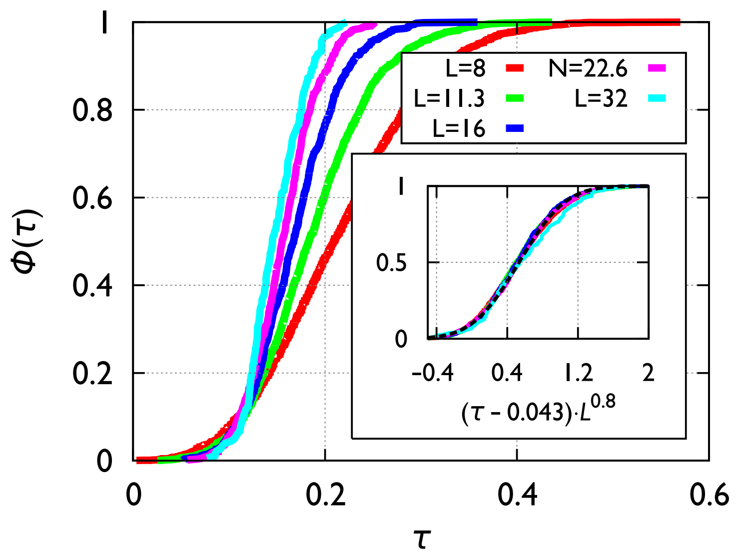


Distribution of stress values at a given strain

- Stresses at a given strain for different realizations are normally distributed
- The standard deviation decreases with increasing system size



$\gamma = 0.05$



$\gamma = 0.2$

Conclusions, outlook, acknowledgements

Conclusions

- The concepts of the stochastic model of Zaiser are in agreement with 2D DDD
- A simple plasticity model of independent avalanches gives a good description of the microplastic part of the stress-strain curves
- A method has been proposed to set the yield threshold distribution in the stochastic model (multiscale modelling)

Acknowledgements

- European Commission, Marie Curie CIG StochPlast 321842
- Hungarian Scientific and Research Fund OTKA PD-105256, OTKA K-105335